Consolidated Quarterly Progress Report No. 3

NOVEMBER 15, 1961

ELECTRONICS RESEARCH LABORATORY

UNIVERSITY OF CALIFORNIA

BERKELEY, CALIFORNIA

CONSOLIDATED QUARTERLY PROGRESS REPORT

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UNIVERSITY OF CALIFORNIA
Berkeley, California

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INTRODUCTION

This progress report contains a review of the research projects conducted within the Electronics Research Laboratory, University of California, Berkeley, and of the progress made during the three month period ending November 15, 1961.

The research work is separated into the areas:

Circuits
Microwave Electronics and Plasmas
Radiation and Propagation
Solid-State Electronics
Systems
Miscellaneous

The research topics within each area are listed alphabetically with regard to the professor in charge of the research.

FACULTY

PROFESSORS	RESEARCH AREA
Bergen, A.R.	Systems
Bevensee, R.M.	Microwave Electronics and Plasmas
	Radiation and Propagation
	Solid-State Electronics
Birdsall, C.K.	Microwave Electronics and Plasmas
Blackwell, D.	Systems
Colgate, S.A.	Microwave Electronics and Plasmas
	Miscellaneous
Dalziel, C.F.	Miscellaneous
Desoer, C.A.	Systems
English, A.C.	Solid-State Electronics
Everhart, T.E.	Circuits
	Microwave Electronics and Plasmas
Gill, A.	Systems
Huskey, H.D.	Systems (Computers)
Jury, E.I.	Systems
Koch, W.H.	Miscellaneous
Kuh, E.S.	Circuits
Pederson, D.O.	Circuits
Polak, E.	Systems
Rumsey, V.H.	Radiation and Propagation
Saunders, R.M.	Miscellaneous
Singer, J.R.	Solid-State Electronics
Sloan, D.H.	Microwave Electronics and Plasmas
Smith, O.J.M.	Systems
Studer, J.J.	Systems
Susskind, C.	Miscellaneous
Thomasian, A.J.	Systems
Trivelpiece, A.W.	Microwave Electronics and Plasmas
Turin, G.L.	Systems
Van Duzer, T.	Microwave Electronics and Plasmas
Wang, S.	Solid-State Electronics
Wattenburg, W.	Systems (Computers)
Welch, W.J.	Radiation and Propagation
Whinnery, J.R.	Microwave Electronics and Plasmas
Woodyard, J.R.	Microwave Electronics and Plasmas
Zadeh, L.A.	Systems

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GOVERNMENT CONTRACTS AND GRANTS

This research was made possible by support from the following organizations:

Contract No.	ERL No.[1]	
AF 49(638)-1043	02	Joint Tri Service support by the Air Force Office of Scientific Research, the Department of the Army Research Office and Signal Corps, and the Office of Naval Research
AF 18(600)-1521	10	Air Force Office of Scientific Research
AF 19(604)-2270	11	Air Force Cambridge Research Laboratories
AF 19(604)-5466	12	Air Force Cambridge Research Laboratories
AF 33(616)-6139	13	Aeronautical Systems Division
AF 41(657)-114	14	Rome Air Development Center
AF 49(638)-102	15	Air Force Office of Scientific Research
AF 33(616)-7553	16	Aeronautical Systems Division
AFOSR-62-70	17	Air Force Office of Scientific Research
DA 36-039 SC-84923	21	U. S. Army Signal Corps
DA 36-039 SC-85278	22	U. S. Army Signal Corps
DA-ARO(D)-31-124-G151	24	U. S. Army Research Office
Nonr-222(53)	30	Office of Naval Research
Nonr-222(57)	32	Office of Naval Research
Nonr-222(92)	33	Office of Naval Research
G-9106	41	National Science Foundation
G-12142	42	National Science Foundation
G-15965	43	National Science Foundation
D-603513	51	Bell Telephone Laboratories
	01	University Sponsored Research

[1]The ERL No. appears as the first two digits of the numbers of each research topic in the lists which follow.

LIST OF REPORTS PUBLISHED DURING THE QUARTER ENDING

15 NOVEMBER 1961

All reports are published in Series 60 of the University of California's Institute of Engineering Research, and are further designated by issue number (as listed below):

No. 394: E.S. Kuh and M. Fukada, "Optimum synthesis of wide band parametric amplifiers and converters," 18 August 1961.
No. 395: S.K. Mitra and R.A. Rohrer, "Synthesis of minimum-pole-sensitive active RC networks," 18 August 1961.
No. 396: E. Polak, "Optimal time control of some pulse-width-modulated sampled-data systems," 22 August 1961.
No. 397: G. August, "Coulomb collisions in strong rf electric fields," 24 August 1961.
No. 398: G. August, "Plasma confinement of electromagnetic waves," 24 August 1961.
No. 399: L.K. Wanlass, "A spin-echo memory for a carrier type digital computer," 28 August 1961.
No. 400: T.E. Everhart, P.L. Morton and D.O. Pederson, "Annual report of research in integrated electronics," 30 June 1961.
No. 401: E.I. Jury, "Final technical report," 30 September 1961.
No. 402: S. Wang and J.R. Singer, "Paramagnetic maser oscillator analysis," 8 September 1961.
No. 403: S.C. Gupta and E.I. Jury, "Statistical study of pulse width modulated control systems, I," 12 September 1961.
No. 404: B. Whalen, "The contraction theorem and its extensions and applications," 12 September 1961.
No. 405: P. Govaerts, "Saturating servomechanism to follow a random process," 14 September 1961.
No. 406: R. Sussman, "The equiangular plane spiral antenna," 15 September 1961.
No. 407: L. Lin, "Optical maser by the method of electron excitation," 27 September 1961.
No. 408: "Notes on system theory, I," 1 October 1961.
No. 409: A.R. Bergen and I.J. Williams, "Verification of Aizerman's conjecture for a class of 3rd order systems," 12 October 1961.
No. 410: I.J. Williams, "Some nonlinear systems satisfying Aizerman's conjecture," 12 October 1961.
No. 411: S.K. Mitra, "A unique synthesis method of transformerless active RC networks," 12 October 1961.
No. 412: A. Gill, "Cascaded finite-state machines," 27 October 1961.
No. 413: A. Gill, "State-identification experiments in finite automata," 13 November 1961.
No. 414: S.C. Gupta and E.I. Jury, "Statistical study of pulse width modulated control systems, II," 13 November 1961.
No. 415: J.W. Carlyle, "Equivalent stochastic sequential machines," 14 November 1961.

CIRCUITS

16-1-01 SOLID CIRCUIT MINIMAL SIZE ANALYSIS. D.A. Waterman (Prof. Everhart)

Work is continuing on the problem of finding the minimum size of a tunnel diode as a function of its physical parameters, such as negative resistance, doping levels, temperature and junction voltage. The method of analysis consists of formulating a mathematical expression for negative resistance in terms of these parameters, and then using this expression to determine the smallest allowable size for any particular value of negative resistance.

An expression has been developed for the tunneling current which contains the integral of an irrational exponential algebraic function. Under the assumption that the fermi distribution functions are essentially the same at absolute zero as at room temperature the algebraic function under the integral reduces to an integrable form. Once the integral is evaluated and expressed in terms of the physical parameters of the system the derivative of the tunneling current with respect to the junction voltage can be found, which will lead to an expression containing negative resistance in terms of these physical parameters.

42-0-03 GAIN BANDWIDTH LIMITATIONS OF ACTIVE DEVICES. J.D. Patterson (Prof. Kuh)

The problem under consideration is that of the gain bandwidth limitations which may exist for an amplifier consisting of an active device imbedded in an arbitrary passive network.

As yet no results have been obtained for the general case of completely arbitrary imbedding. For one-port active devices consisting of a lossless network terminated in a negative resistance the gain bandwidth limitation and the required imbedding network can be found. It was noted that this type of active device may be unstable under all passive imbedding and thus not useful as an amplifier.

At present we are considering the special case of cascade amplifiers, i.e., amplifiers consisting of two-port active devices in a cascade configuration with passive interstage networks. For unilateral active two-ports the gain bandwidth limitation can be found. Work is now in progress on the problem of non-unilateral devices.

42-1-04 A STUDY OF "TUNNEL DIODE-RC" NETWORKS. S.K. Mitra (Prof. Kuh)

The necessary and sufficient conditions for the realization of driving-point immittances of such networks (assuming a parallel connection of a positive capacitance and a negative conductance, as the equivalent circuit of a tunnel diode) containing n tunnel diodes has been obtained. Work is continuing on the transfer function synthesis of such networks with a minimum number of tunnel diodes.

42-1-05 WIDE BAND PARAMETRIC AMPLIFIERS STUDY. M. Fukada (Prof. Kuh)

In a previous report, Series No. 60, Issue No. 394, the theoretical limitations on gain and bandwidth for parametric amplifiers have been derived and a straightforward design procedure to approach the optimum situation has been developed. The above treatment, however, is restricted to the case where the variable capacitance diode is lossless and is resonated in shunt. In this research possible extension of the theory to the series diode case and to the amplifiers with the diode dissipation have been studied.

The bandwidth for a series diode case is related to the corresponding shunt diode case by a simple formula:

$$B_{series} = \frac{2\left(\frac{\omega_o'}{\omega_o}\right)}{1 + \left(\frac{\omega_o'}{\omega_o}\right)^2} B_{shunt},$$

where B_{series} and B_{shunt} are the bandwidths for the series case and shunt case, respectively. ω_o and ω_o' are the center frequencies of the signal and the idler frequencies, respectively. By using this relation and the previous results for the shunt diode case the theoretical limitations on bandwidth for the series diode case are derived.

The transducer power gain G_a of a shunt diode amplifier with the diode dissipation is given by

$$\sqrt{G_a} \approx \frac{1 - d}{2|\rho|\left(1 - \frac{d + d'}{2}\right)}$$

where ρ is the reflection coefficient of a simple network. d and d' are normalized dissipation factors of the diode at ω_o and ω_o', respectively. A modified design procedure for the shunt diode amplifier has been devised.

42-1-06 SYNTHESIS OF ACTIVE RC n-PORT NETWORKS. S.K. Mitra (Prof. Kuh)

Our previous work, Synthesis of Minimum-Pole-Sensitive Active RC Networks, has been generalized to the case of n-ports. The synthesis is based on RC-LC transformation of a given RC impedance matrix [Z] of order n and identifying the transformed impedance matrix [$\hat{Z}$] as the odd part of a p.r. matrix [G] (which we will designate as the Corresponding matrix to [Z]).

We have shown that if the corresponding matrix [G] is such that the even part of [G] is of rank k, then we can synthesize [Z] as k n-ports connected in series, where each of the n-ports consists of a (n + 1) port LC network N terminated at its (n + 1)th port by a negative LC impedance of value equal to the reciprocal of the open-circuit driving-point impedance of N at the (n + 1)th port. The (n + 1) port LC network can be synthesized by the usual methods and as a result may use transformers. Performing an inverse LC-RC transformation on the n-port active LC network, we obtain an active RC realization of [Z].

The dual result holds for the case of admittance matrices.

42-1-07 A UNIQUE SYNTHESIS METHOD OF TRANSFORMERLESS ACTIVE RC NETWORKS. S.K. Mitra (Prof. Kuh)

A unique decomposition of active RC driving-point impedance functions has been obtained by considering the driving-point synthesis problem in terms of the reflection coefficient. Application of the decomposition guarantees the realization of the driving-point impedance in practical network structures containing one negative impedance converter. The method imposes no restriction on the impedance function, except that it has only to be a real rational function. The synthesis technique is adaptable to digital computers.

An Electronics Research Laboratory report on this research project is under preparation.

02-1-04 TIME VARYING RC NETWORKS. A. Paige (Prof. Kuh)

This investigation is concerned with determining the characteristics of networks constructed of elements which may be functions of time. The immediate problem is concerned with the time varying analog of the driving-point impedance of an RC network. Specifically, the network is assumed to be made up of time varying resistances and capacitances which are 1) always positive; 2) all elements are periodic with the same period.

The investigation has centered about finding an equivalent network for driving-point functions similar to that of Foster canonic forms in time invariant networks. One such network has been found but it is not known if it is general enough.

One problem which has received considerable attention is that of stability of such time varying networks. Certain conditions which assure stability have been determined but they are not completely general.

A problem which is related to that of stability is the one of determining if time varying RC networks are capable of amplification, and if so, under what conditions.

The primary emphasis of this project for the immediate future will be that of attempting to resolve the question of stability of these networks.

02-1-29 PIECEWISE CONSTANT TIME VARYING NETWORKS. R.A. Rohrer (Prof. Kuh)

As a preliminary to the full scale investigation, methods of analyzing and synthesizing networks containing a periodically operated switch are being considered. A frequency-domain analysis method now being formulated promises to yield results useful in the synthesis of such networks when the steady-state response is specified.

02-1-05 TUNNEL-DIODE OSCILLATOR STUDIES. R.S. Pepper (Prof. Pederson)

In this topic the optimum or limiting performance of a simple tunnel-diode oscillator configuration was studied. This study gave the conditions for obtaining the minimum period of oscillation as well as the conditions for obtaining the maximum fundamental component of output power at a given frequency for a tunnel-diode oscillator having an R-L load. This work has been completed and is reported in a technical report, Series No. 60, Issue No. 388, entitled "The Minimum Period of Oscillation of Simple Tunnel Diode Oscillators" and in a Ph.D. dissertation by R.S. Pepper, Department of Electrical Engineering.

Present work in this field centers on a tunnel-diode oscillator with a parallel RC load. For this circuit the minimum period of harmonic or near harmonic oscillation is of interest. It appears possible to achieve a smaller period of near harmonic oscillation with this configuration than with the circuit previously studied. In addition, the available output power at the fundamental frequency will be compared to that obtainable from the other circuit.

02-1-06 HARMONIC OSCILLATORS. D.K. Lynn (Prof. Pederson)

The object of this study is to find for certain electronic oscillators second order nonlinear differential equations which approximate the exact higher order equation of the oscillator. Once the second order equation is found information as to period, amplitude and wave shape can be found by known techniques. At present two examples are being considered. The first is a simplified transistor Colpitts oscillator; the second is a tunnel-diode oscillator with a parallel RC load. Both circuits are described by a third order differential equation.

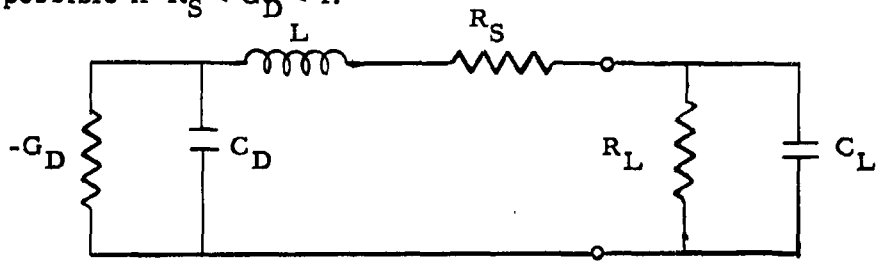
The approximating equation may be of the following form:

$$\ddot{v} + a_1(v)\,\dot{v} + a_0(v)\,v = 0 \qquad (1)$$

If $a_0(v)$ is a constant, i.e., $a_0(v) = a_0$, the equation is called the generalized Van der Pol equation. At present it appears that the Colpitts oscillator can be approximated in this manner. The more general case describes a tunnel diode with a series RL load. In addition, it appears that the tunnel diode oscillator with an RC load can be approximated with (1).

For the tunnel-diode oscillator with an RC load approximate solutions of the third order equation are being made using piecewise linear techniques with a computer and using a perturbation technique. Both of these methods have been used previously for the tunnel-diode oscillator with a series RL load.

As a preliminary step in solving the tunnel diode oscillator equation, a linear analysis has been made to investigate the conditions for near-harmonic oscillation. The linear analysis shows that when the load is adjusted to give two natural frequencies on the jω axis, there is the possibility that the third natural frequency may lie in the RHP. This would result in non-harmonic oscillations. Therefore, care must be taken to ensure that the third natural frequency lies in the LHP. It appears that when the third natural frequency is at the origin the maximum frequency of harmonic oscillation is obtained. This frequency is higher than the harmonic oscillation frequency with a resistive load but is smaller than the resistive cutoff frequency of the tunnel diode. If the circuit model of the oscillator shown in the figure is normalized for $L = C_D = 1$, harmonic oscillations are possible if $R_S < G_D < 1$.



A natural frequency at the origin is obtained for $R_L = \frac{1}{G_D} - R_S$.

For

$$C_L = \frac{G_D}{\left(G_D - R_S\right)\left(1 - R_S G_D\right)}$$

the other two natural frequencies lie on the jω axis and harmonic oscillations result at a frequency:

$$\omega_{RC} = \left[1 - G_D^2 + \left(1 - \frac{R_S}{G_D}\right)\right]^{1/2}$$

As a comparison for a pure resistive load ($C_L = 0$), and for $R_L = G_D - R_S$ the circuit oscillates harmonically at a frequency

$$\omega_R = \left(1 - G_D^2\right)^{1/2}$$

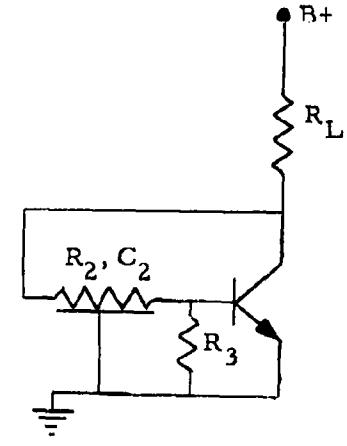
The resistive cutoff frequency of the tunnel diode is

$$\omega = \left(\frac{G_D}{R_S} - G_D^2\right)^{1/2}$$

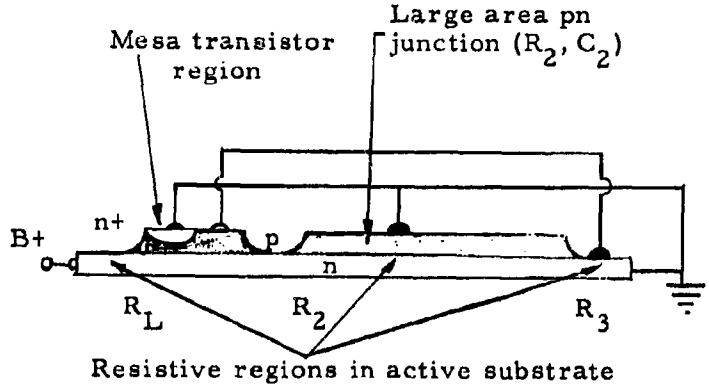
16-1-02 HARMONIC OSCILLATION IN A SOLID. G. Hachtel (Prof. Pederson)

In this project we seek the necessary conditions to achieve near-harmonic oscillation in a semiconductor integrated circuit. Much information is available on oscillations in lumped circuits, e.g., gain-phase conditions, negative resistance, etc. For an integrated circuit the necessary conditions to be most useful should be in the terms of charge flow and charge control patterns and time variations of stored charge.

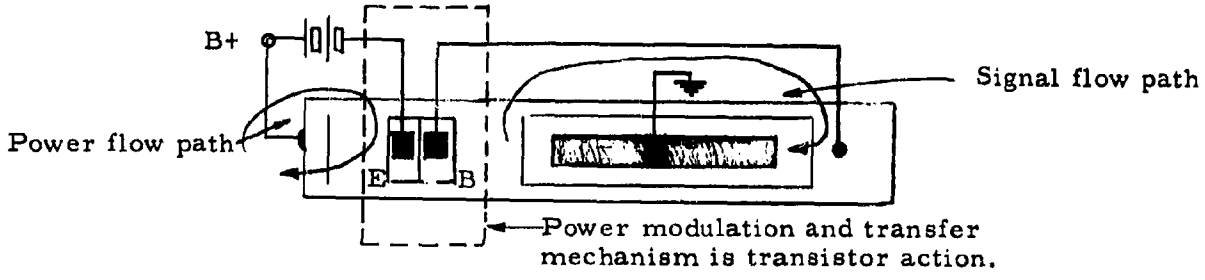
We have found it convenient to specify three basic components of an oscillator --the signal flow path, the power flow path and the power modulator and transfer mechanism. These components are illustrated in Figure 1 for the phase shift oscillator that we have reported on previously. At the present time attention is centered on another possible integrated oscillator, one utilizing a unijunction transistor (UJT) as shown in Figure 2. By working with and comparing these oscillators arising from different lumped models, it may be possible to establish the desired basic conditions for oscillation.



(a) Circuit Model



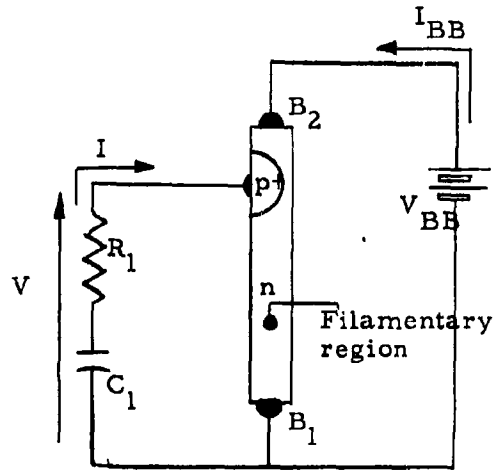
(b) Integrated Realization



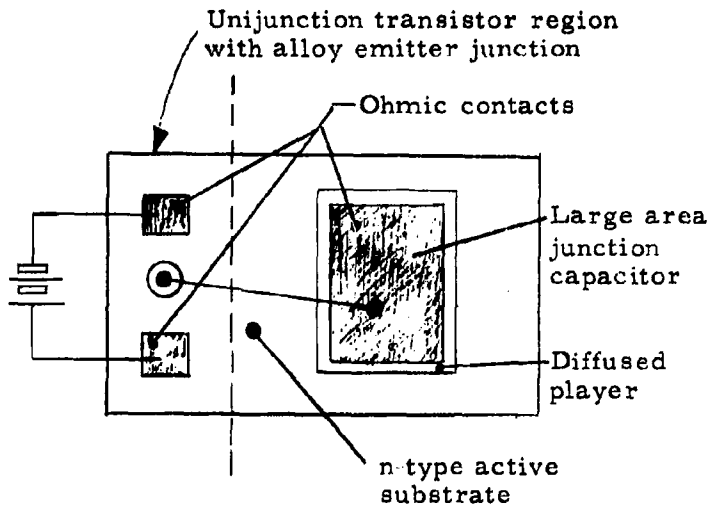
(c) Descriptive Representation

Figure 1. Integrated Phase Shift Oscillator Realization

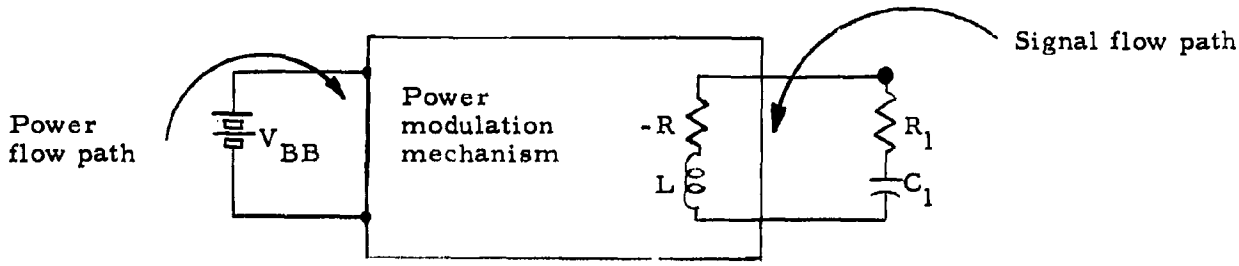
During the past quarter studies have been made to determine for the UJT oscillator physical explanations for and interrelations of the inductive effect of the delayed charge variation, the conversion mechanism that produces the negative resistance (conductivity modulation) and the current flow between the oscillator capacitance, C, and the UJT during a period of oscillation. Relations have been found between the negative resistance and the physical parameters of the device mechanism of the UJT and bias conditions. The delay phenomenon which produces the equivalent inductive effect is now being investigated.



(a) Parallel Resonant Oscillator Circuit



(b) Integrated Realization



(c) Descriptive Representation

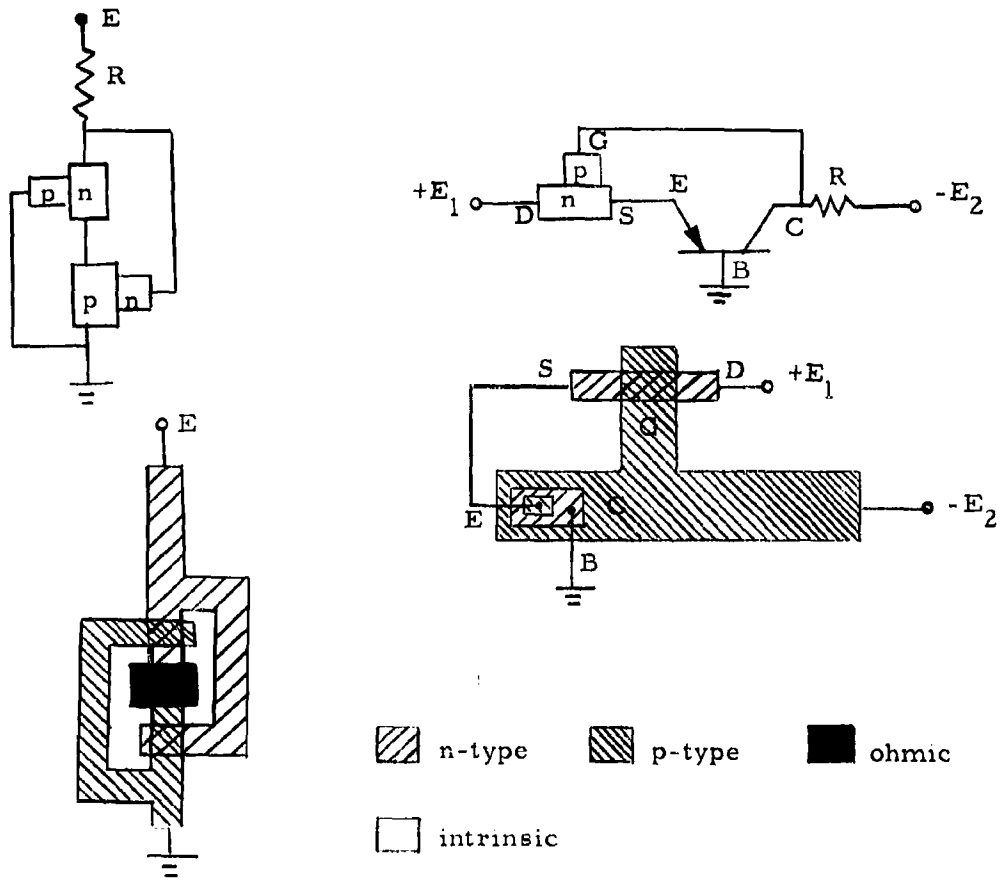
Figure 2. Parallel Resonant Oscillator Realization

16-1-04 BISTABILITY IN A SEMICONDUCTOR SOLID. L.O. Hill (Prof. Pederson)

In our previous work with integrated bistable circuits it became apparent that a unified approach was necessary to investigate and determine possible bistability of a lumped or an integrated circuit. A controlled resistance circuit model has been proposed which provides this unified approach. The model characterizes the operation of most active devices or active device mechanisms but is still independent of the physical phenomena responsible for the operation of any particular device.

From the systematic use of the controlled resistance model a very large number of possible bistable circuits can be generated. Among these are the familiar negative resistance circuits such as those using the 4-layer switch as well as cross coupled circuits such as the Ecles-Jordan circuit. By including the insight gained by a study of semiconductor realization techniques it is possible to choose possible circuit configurations which are novel and which show promise for integrated realization. Two basic circuits using bipolar and/or unipolar transistors to supply the active device mechanism are shown in the figure together with a planar view of possible integrated realizations.

A technical report covering this topic is now being written.



16-1-06 INTEGRATED REALIZATIONS OF BANDPASS AMPLIFIERS. D.A. Hodges (Prof. Pederson)

During this quarter experimental studies have been conducted on a lumped analog of the integrated bandpass amplifier proposed in earlier theoretical studies. The circuit under consideration is shown in Figure 1.

The combination R_f - C_f is a distributed resistance-capacitance which can be realized as a back-biased large area pn junction. R_f, C_f and R_s can be adjusted to give zero or positive feedback in the desired passband and negative feedback at other frequencies, thus yielding a bandpass transfer characteristic for the entire circuit. The Darlington pair is used to give the gain necessary for a narrow passband with reasonable stability of gain.

A possible realization of this circuit in integrated planar form is shown in Figure 2. An intrinsic block of silicon is doped first with n-type impurities, then p-type, by diffusion methods, over the indicated areas. The emitters of the transistors might be formed by alloying. External connections and internal interconnections may be made using conventional techniques.

A field effect transistor may be connected to yield the R_f - C_f combination as shown in Figure 3. When properly biased this device has a larger voltage transfer ratio than a passive formulation of R_f and C_f, due to the modulation of the resistance R_f in the field effect transistor. This property can narrow the passband of the complete amplifier.

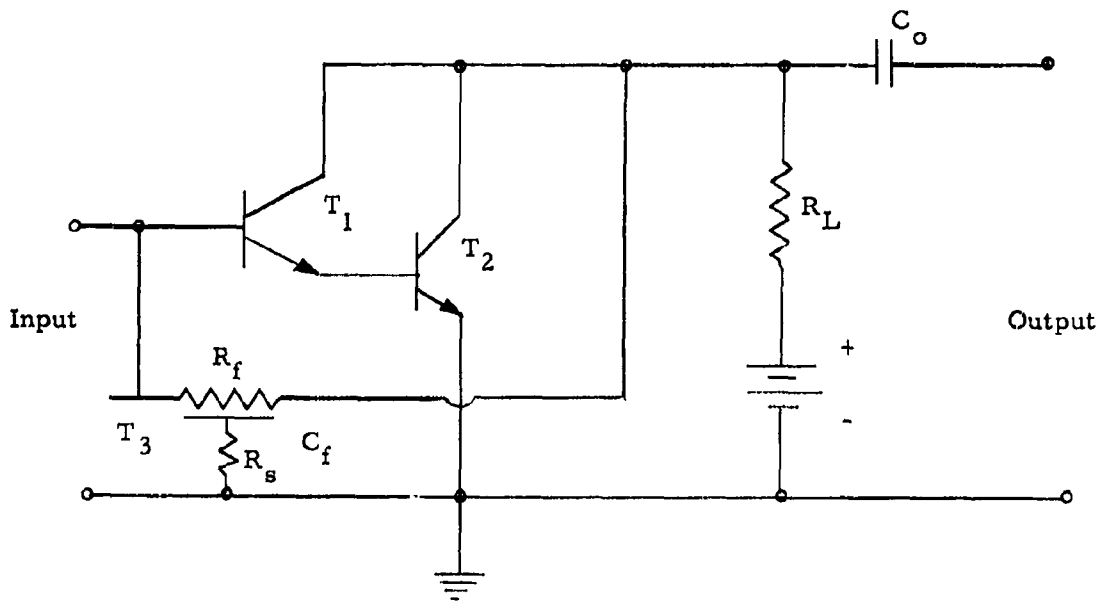


Figure 1

Experiments have been conducted on a lumped circuit similar to that of Figure 1, using a field effect transistor in the feedback path. (Arrangements were made to enable independent adjustment of biases on T_1, T_2 and T_3.) As R_s is reduced oscillation is approached and the passband may be made arbitrarily small. A reasonably stable amplifier with a Q of 10 and power gain of over 20 db has been obtained at a center frequency of 5 mcs.

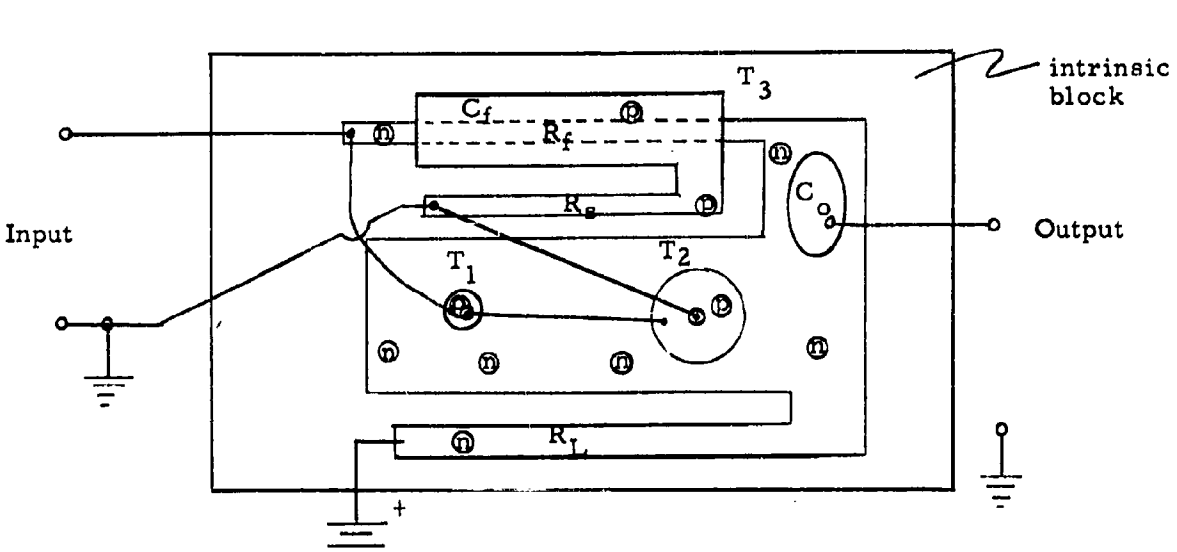


Figure 2

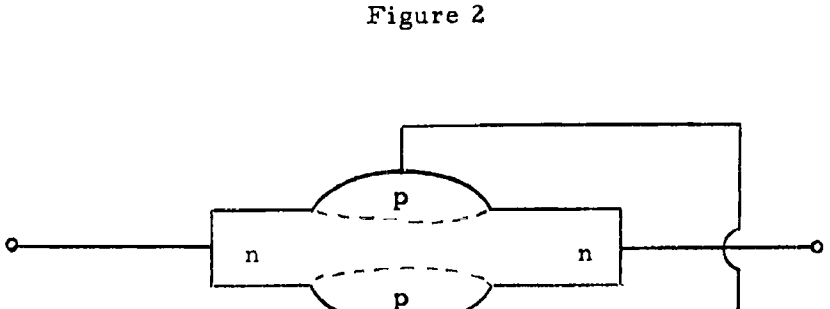
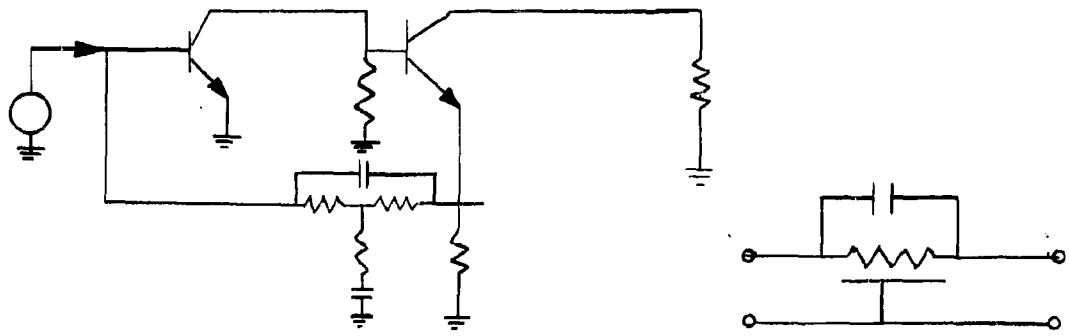


Figure 3

02-1-28 ANALYSIS OF INTEGRATED BANDPASS AMPLIFIERS.
A. Thiele and K. Y. Tsai (Prof. Pederson)

Bandpass amplifiers can be realized without using inductance by the use of feedback amplifier configurations or by other active RC synthesis techniques. These amplifiers in lumped form can often serve as prototypes for integrated circuit realizations. In this project analyses and experiments are being made with active RC circuits with particular emphasis on problems or aspects which will arise when an integrated amplifier is to be made.

For a feedback amplifier, such as shown in the figure, a design procedure has previously been developed for low pass amplifiers using the proper location of "phantom zeros" (i.e., zeros of the feedback function), c.f. technical reports, Series No. 60, Issues No. 274 and 307. Our present work is to extend and optimize this procedure to the bandpass situation as well as to investigate the incorporation of distributed RC networks into the



feedback path. For the given amplifier configuration it appears that the angle of approach of the root loci to the phantom zeros should be approximately 0^{o} for best desensitivity of passband shape to transistor gain variation. Another analysis is in progress to determine the best location of the poles of the gain function of the basic amplifier and of the feedback function in relation to the location of the phantom zeros. For distributed RC networks, such as shown in the figure, dominant pole-zero descriptions are being investigated in order to use simply the root locus methods of analysis and design.

MICROWAVE ELECTRONICS AND PLASMAS

01-1-43 STAGGER-TUNED CAVITY CHAIN BEAM AMPLIFIER. Y. Satoda (Prof. Bevensee)

The beam and circuit equations for a 5-cavity amplifier terminated with an output load have been programmed for the IBM 704 computer. They will be solved during the next quarter in order to design a practical structure with high gain and good bandwidth in S-band.

01-1-44 EQUIVALENT CIRCUITS FOR WAVEGUIDE OBSTACLES. D. Trima (Prof. Bevensee)

Equivalent circuits will be developed for the dominant TM waveguide mode impinging on various obstacles in rectangular and circular waveguide. This work will augment the results presented for TE waveguide modes in Chapter 5 of the Waveguide Handbook by Marcuvitz. The circuits will be derived from a variational expression for the obstacle susceptance.

01-1-45 GROUP THEORY INVESTIGATIONS. (Prof. Bevensee)

Examples of the use of group theory principles for solving problems in circuit theory, field theory and transmission theory have been developed. These will be published in a report.

13-9-01 FAST-WAVE INTERACTION. L. Haas (Prof. Birdsall)

Objective: Investigation of microwave generation and amplification by fast-wave interaction using no magnetic field.

Analysis and experimental evidence have shown that because of transit time dispersion of the electron stream the interaction in the Multiflex Traveling-Wave Tube is inefficient with a maximum efficiency to date of 0.06 per cent. In order to overcome this defect of the M.T.W.T. a different scheme of repeller focusing has been proposed. The scheme that has appeared most promising is a double period focusing in which a slow electron in one period is a fast electron in the next. This is accomplished ideally by repellers made of sections of spheres. Compensation must be made for the fringing fields, however. Unfortunately, the undesirable focusing and defocusing effects of these fields cannot be removed for all electrons in the stream. As a result, the interaction is again expected to be inefficient; but still, the efficiency should be quite a bit greater than in the M.T.W.T.

A tube utilizing double period focusing is now under construction. Since the bunching parameter does not remain the same for succeeding interactions the transmission line analog analysis of the M.T.W.T. obtained earlier does not apply. A theory has been worked out for this type of interaction, but since the analysis predicts zero gain at waveguide cutoff and maximum gain elsewhere, rather than a maximum gain at cutoff, its validity is still in doubt. This matter will be corrected and the tube construction will soon be completed.

13-0-11 FAST-WAVE INTERACTION. R.E. Lundgren (Prof. Birdsall)

Objective: The generation and amplification of microwave signals by devices using fast-wave interaction is under investigation.

Analysis: We are currently studying the interaction mechanism of the helical-beam device described in the Consolidated Quarterly No. 1. A program for the IBM 704 computer was written to analyze a simple model of this interaction. This model assumes that the interaction field is independent of radius, that the transit angle of the interaction is negligible and that the interaction produces only a modulation of the tangential velocity of the electrons, thereby neglecting the attendant change in their radial position. Thus the model assumes the interaction to be like that which would be produced by an infinitesimal gridded gap. Some preliminary calculations based on this simple model have indicated an efficiency of at least 33% for a tube employing 16 traversals of the beam past the waveguide gap. Our next task will be to include the modulation of radial position in the calculations since this is a first-order effect comparable to the tangential velocity modulation. Following this we should then include the radial variation of the interaction field.

13-1-01 ELECTRON STREAM INSTABILITIES. J. Frey (Prof. Birdsall)

Objective: To study instabilities in electron (or ion) streams in drift tubes, initially to obtain limiting current for a drift tube of finite length.

Potential profiles for a drift tube of finite length (which also may be thought of as a diode with an enclosing pipe) have been obtained for a number of stream currents. Limiting current has not been reached as yet, even at currents very much larger than can be supported in a one-dimensional diode.

13-0-10 SLOW-WAVE CIRCUITS FOR MILLIMETER WAVELENGTH TUBES. D.E. Chaffee (Prof. Birdsall)

Objective: 1) To obtain slow-wave circuits for use at millimeter wavelengths in electron-stream amplifiers and oscillators, initially to investigate "wrapped-up" or circular ladder circuits with large ka, which can be used with a relatively large-diameter hollow electron stream; 2) to build a tube using these circuits.

The construction of the fine millimeter wavelength tube is continuing.

13-0-14 ELECTROMAGNETIC WAVE PROPAGATION IN PLASMAS. V. Bevc (Prof. Everhart)

This research has been completed and a technical report has been issued entitled "Fast Waves in Plasma-Filled Waveguides," Series No. 60, Issue No. 362.

02-1-37 EXPERIMENTAL INVESTIGATION OF MICROWAVE PROPAGATION IN A PLASMAGUIDE. R.N. Carlile (Prof. Everhart)

Objective: To determine experimentally the phase characteristics and field configurations of the modes of propagation in a cylindrical waveguide partially filled with an ion neutralized plasma (plasmaguide) in the presence of a finite magnetic field parallel to the waveguide axis. The modes to be investigated are those which have 1) no azimuthal variation; 2) have an azimuthal variation of period 2π. Measurement of fast waves $v/c \gtrsim 1$ will be emphasized.

A large portion of the past quarter was given to the design, construction and assembly of equipment that could be used to determine the feasibility of using a Hg discharge in a system that will resonate in modes derived from the forward wave and cyclotron modes of a plasmaguide. If this feasibility can be demonstrated in a resonator at fixed length, which is relatively easy to build, then a resonator of variable length will be constructed.

Tests have begun on a resonator (plasmacavity) where the continuation of the end walls across the plasma take the form of the anode of the discharge and a screen, respectively. The screen is extremely coarse, consisting merely of two 10 mil tungsten crossed wires.

Initial attempts to obtain a resonance of the forward wave plasmaguide mode have been thwarted by the ineffectiveness of the screen in reflecting energy. The VSWR of the screen, measured in the usual way of probing for maxima and minima, is only .5 (over a frequency range of 1.5 -2.0 kmc). This may represent true loss of energy through the screen, or the field minima may be masked by noise fluctuations in the charge density or by the presence of other modes of the system. If the former is the case, then an obvious solution would be to use a finer mesh screen.

The existing system is sufficient to measure plasma frequency and collision frequency. With a knowledge of these parameters one can determine which modes will have sufficient amplitude to be measured.

11-1-01 PLASMA CATHODE. M. Chamran, Y. Ikeda (Prof. Sloan and Susskind)

The objective of this project is the study of the emission and interaction processes in gaseous crossed-field configurations and the applications of the results to the design of high power microwave devices.

The general nature of both the plasma beam and the required fast wave interaction forward wave structure are now understood.

The copper cathode has insufficient secondary emission to operate without electrons from gas ionization. If the load impedance is a little too low, additional gas can bring good operation again. The beam velocity for easiest starting was found to be the same with many cathode materials and dimensions and is approximately that velocity for which the various electrons have ionization collision cross sections of the greatest average value.

B and the distance d between electrodes must be great enough to hold a reservoir of electrons near the cathode, sufficient for backbombardment. Less Bd is required with greater secondary emission ratio δ. An intolerably hot thermionic cathode, with no need for secondary emission, could give the greatest current and beam velocity. In all cases the maximum intensity of random diocotron waves ω_m occurs for wavelengths about 2d. The shortest diocotron wave limit occurs at $\omega = \omega_c$ of cyclotron motion, varying from several times to many times greater than ω_m. Any TEM resonance in the interelectrode space will be weakly excited if its frequency lies in this diocotron range, between ω_c and about $\omega_c/100$, forming a line spectrum superposed on noise.

The space-charge-free drift velocity $v = E/B$ is about twice the velocity of bunches synchronized by a slow wave structure of pitch $p = d$, matching the half wavelength of maximum random waves. This rapid passage of electrons "up, over the top and down the front" of a bunch will be relied upon to sustain bunching in the proposed helical beam for a doppler magnetron.

A helical beam which may move along the axis at a velocity about as great as v, may be given the required axial thrust by any or all of the three following methods. An axial current and surrounding B' adds to axial B to form helical flux which makes a helical beam. A strip of helical ridge on the cathode with end hat action produces a similar effect. The greatest axial velocity, however, comes from a slight twist of the slow wave structure. This latter method gives greatest axial velocity to electrons nearest the anode, but this tends to destroy bunches. The "up and over" motion would be greatest for electrons farthest from the bunch and should be able to prevent this debunching.

In the doppler magnetron electrons interact with a sequence of axial water cooled tubes in the usual magnetron manner. In the helical beam the encircling group of electrons have an axial velocity which is synchronized with the group velocity of a wave traveling axially along the highly dispersive anode structure. Within the traveling group the circumferential phase velocity is infinite as in stationary magnetron oscillators, but the frequency within the traveling group is related to a stationary observer by the doppler effect. This is a forward wave, fast-wave interaction capable of great power. The axial line impedance is less than 10 ohms, so the question now is whether the helical form of our plasma can be strong enough to operate such an amplifier. The beam may be outside or inside of the anode. The latter offers opportunity for many output seals to carry high power.

To study the high current diocotron oscillations of the plasma beam will require the addition of a hard tube modulator in the 20 KV d-c supply line. For several years operation has been limited to d-c observations because the pulse line voltage cannot be regulated closely enough. Several thousand amperes are now needed. A plasma beam tube could probably be developed in a year or so to do the modulating. However, parallel operation of many of an existing type of hard modulator tube is preferred to expedite study of the helical beam for amplifier use.

Now that the thick electron sheath and thin plasma is fairly well understood work is being resumed on equipment to investigate thin sheaths with thick plasma cathodes, in which ion paths are magnetically curved to prevent direct passage from anode electron sheath to cathode metal.

11-1-03 DOUBLE-STREAM CYCLOTRON WAVE INTERACTIONS. B. J. Maxum (Prof. Trivelpiece)

The objective of this project is the study of the interactions between drifting electron streams in a finite magnetic field.

The nonconvective instabilities previously predicted on the basis of an analysis for the completely filled waveguide have been further analyzed for the nonfilled case. Computations have been carried out for the coaxial configuration of the experiment. On the bases of these computations it has been found that ϕ independent modes have a drastically reduced coupling as compared with the filled waveguide case. It has been established that the potential function $\phi(r)$ must pass through zero inside of the solid (inner) stream and near the outer edge of that stream. This results in most of the kinetic power to be carried in the inner stream and results in very weak coupling to the outer stream. The first ϕ dependent mode, however, was found to give substantial coupling and, consequently, the probe configuration of the experiment has been changed to better sense this mode.

The computer program is being generalized to investigate the characteristics of the complex roots which give rise to the nonconvective instabilities.

The experiment is in progress.

13-1-03 INTERACTION OF A DRIFTING ELECTRON STREAM WITH THE BACKWARD-WAVE MODE OF PROPAGATION IN A FERRITE ROD. J. Spector (Prof. Trivelpiece)

The objective of this project is to investigate theoretically and experimentally the interaction between a drifting electron stream and the magnetostatic modes in longitudinally magnetized ferrite.

During the preceding quarter cold tests were performed on the interaction circuit. An effort was made to eliminate the r.f. leakage fields which had plagued previous experiments. This was done and it was found that with the original gold coating used in the last "hot" experiment there was no r.f. transmission. With the gold coating removed r.f. transmission was observed and the total attenuation varied between 60 and 80 db as a function of frequency and magnetic field. This is in rough agreement with what theory predicts.

In the next quarter cold testing will be continued so as to find a suitable conductive coating for the yttrium-iron garnet. Following this the tube will be reassembled and hot testing will resume.

22-1-01 SHIELDED-GUN CROSSED-FIELD AMPLIFIER. R.A. Rao (Prof. Whinnery)

Previous experiments indicate that the excess noise in crossed-field amplifiers may be largely due to phenomena taking place in the potential minimum region. The purpose of this project is to verify the importance of the crossed-field potential minimum and to develop a low-noise crossed-field amplifier. This will be done by magnetically shielding the gun region of the amplifier, thus preventing the formation of an M-type potential minimum. Then the noise figure of such a shielded-gun amplifier will be measured and compared with available noise figures of conventional crossed-field amplifiers. The effect of O-type noise-reducing schemes in the gun will also be studied.

As a first step, the effect on the electron trajectories of the non-uniform magnetic field in the vicinity of the shielding box was studied using the Bendix G-15 computer. On the basis of these data, a dc model of the amplifier was designed.

The tube is now ready and being tested. The results of the tests will be reported in the next quarterly report. Mainly, the focusing of the beam and its transmission through the interaction region will be studied. The electrode potentials will be optimized for maximum transmission of the beam to the collector. The data obtained will be used for the design of the rf model.

22-1-02 POTENTIAL MINIMUM ANALYSIS. M.A. Pollack (Prof. Whinnery)

The objective of this project is the study of the effects of the low-velocity region, and the potential minimum in particular, on noise transport in the crossed-field diode.

It has been found that the potential minimum fluctuations calculated by the Monte Carlo method are dependent on the average number of "electron"-sheets filling the diode. An estimation of this dependence has been made and will be used to normalize the spectrum of the fluctuation. Computer programs have been prepared to evaluate the self- and cross-power density spectra of current and velocity fluctuations at the cathode, anode and potential minimum planes.

An experiment has been designed to verify Van Duzer's Transformation of Fluctuations theory[1] and to check on assumptions about noise quantities near the potential minimum. A crossed-field space-charge-limited diode is being constructed with a gridded gap at the anode to measure convection current fluctuations. This noise current will be measured as a function of the various diode parameters and compared to the noise current without the crossed magnetic field.

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[1] "Transformation of Fluctuations Along Accelerating Crossed-Field Beams," T. Van Duzer, IRE Transactions on Electron Devices, Vol. ED-8, No. 1 (January 1961).

22-1-03 NOISE FIGURE MEASUREMENTS AND CALCULATIONS. M.N. Raju (Prof. Van Duzer)

This research has been just taken up and it is proposed to continue the noise figure measurements reported earlier by J.F. Rando. It has been observed that under certain conditions crossed-field amplifiers become very noisy. This phenomenon has been studied by many investigators and one of the interesting observations is the presence of electrons with potential energies less than the cathode potential. These electrons can be collected by the sole electrode when the sole is negative with respect to the cathode. Further investigations by Miller have shown that the noise figure depends upon the d.c. condition in the gun region. When the value of anode voltage over magnetic field squared is less than a critical value the noise appears. Under normal operating conditions this ratio is well above the range in which noise figures are predicted to be high. This phenomenon should then play a small role in determining the noise figure of the crossed-field amplifiers. In the absence of any noise sources, it is guessed that the noise is mainly caused by the ordinary shot noise and Rack velocity fluctuation. Based on this, Rando has derived an expression for noise figure in terms of various parameters. In order to verify this expression it is proposed to vary each of these parameters and measure the resulting noise figure to see whether it agrees with the theoretical value. Unfortunately, these parameters cannot be varied over a wide range as any such variation disturbs the model assumed in deriving the noise figure expression.

22-0-06 NOISE FIGURE MEASUREMENTS AND CALCULATIONS. J.F. Rando (Prof. Whinnery)

A technical report is under preparation. The research is summarized as follows.

The noise figures of forward and backward-wave crossed-field amplifiers are derived. The velocity fluctuations at the cathode are transformed to the slow-wave structure using a modification of the Llewellyn-Peterson equations. Matching the boundary conditions between the beam and the circuit, the magnitudes of the waves excited are obtained. Noise figure measurements were made using a backward-wave amplifier. Since poor agreement is found between the theoretical and experimental gain, it is perhaps premature to compare the noise figures. The comparisons made showed that the experimental noise figures were several db lower than the theoretical values for the corresponding cases.

13-0-03 PLASMA MACHINE PROJECT. D.B. Cummings (Profs. Birdsall, Colgate and Trivelpiece)

Objective: To construct a magnetic mirror configuration to contain a high-temperature plasma (ion energies on the order of 1000 ev) for a useful time; to perform experiments with this hot plasma which will increase knowledge of stability of containment means, of plasma compression, heating and transport means, of plasma dynamical behavior and of energy recovery means.

A. Vacuum: Parts of the epoxy-fibreglas vacuum chamber were finished, the chamber assembled and checked with the helium leak detector. Several leaks were found and repaired. The chamber was set up in an oven and baked at about 100°C. A mechanical pump was used with a liquid nitrogen trap to pump condensable hydrocarbon vapors from the hot epoxy. There is indication that the pumping speed to the cold trap must be increased.

The rack to hold the finished vacuum chamber and pump system is assembled. Work on the modified high-vacuum system pump is almost finished. The upper jets have been remade from stainless steel. The bottom jet must be made smaller to change the fluid flow in order to decrease radial heat transfer. The fore-vacuum assembly for this system is finished and in place.

B. Plasma sources: Design of the plasma sources has been completed, drawings are prepared and parts are being made in the shop or being purchased. The modified cold trap for the vacuum system is being made.

C. Mirror coils: Detailed design of the coils has begun and continues. Parts will be ordered and fabrication begun. A mock-up of the final stage is being made to check calculations.

D. Electronics: The capacitor rack has been made and installed. However, bracing to the building wall must be reworked. More parts for the capacitor bank and control system have been ordered and others received. Prototypes of the ignitron mounts and of the coaxial isolation ignitron firing pulse transformer are being made. The coaxial ground loop inductor and the signal monitoring safety box are being made. Installation of much of the equipment can begin after the capacitors are in place in the rack and have been tested.

11-1-04 INVESTIGATION OF A BEAM PRODUCED PLASMA. C.E. Kuivinen (Prof. Trivelpiece)

The purpose of the project is to investigate the interaction of an electron beam and the E-M waves in the plasma formed by firing the beam into a neutral gas.

During the past quarter a high Q cavity was built, and the perturbation technique was used to determine the plasma density (n_p) created by an electron beam passing through a neutral gas. Data was obtained for a wide range of parameters, the most important being that of n_p versus neutral gas pressure (P). n_p was found to be proportional to P.

Once the plasma density was determined the plasma column was put in a metal cylinder, thus allowing propagation of E-M waves. This system supported forward wave oscillations over a fairly wide range of frequencies. It is hoped that with a slight modification of the input-output arrangement backward wave interaction will be observed.

The data accumulated will be included in the final report.

11-0-05 INTRINSIC OSCILLATIONS IN GASEOUS CONDUCTORS.
B.E. Dobratz (Prof. Woodyard)

The title of this project (formerly "Investigation of Oscillations in Gaseous Conductors Near the Threshold of Conduction") has been altered in keeping with an increase in the scope of the investigation. With larger electrode spacings it has been found that near-threshold conditions are not necessary.

Work of the previous quarter concentrated on study of optical striation phenomena in mercury vapor and hydrogen. "Positive" striations moving from the anode toward the cathode were observed in mercury vapor and distinct stationary striations were observed in hydrogen.

This quarter's work began with examination of the mercury vapor device under an applied a.c. voltage. The existence of moving positive d.c. striations was further substantiated by observation of stationary a.c. striations. By adjustment of the a.c. voltage amplitude, such striations could be achieved for frequencies from 30 c.p.s. to 10,000 c.p.s., the range of the equipment used.

Attempts were made to duplicate these observations in ultrapure helium in a tube of very similar geometric dimensions. For all pressures of helium from approximately 10^{-5} mm. of mercury to a few mm. of mercury, no oscillations were observed by all previous methods of observation. Introduction of a small percentage of hydrogen into the helium atmosphere produced oscillations of frequency typical of hydrogen alone. These oscillations ceased upon removal of hydrogen from the tube.

The next quarter's work is planned to include study of a pure hydrogen atmosphere for striation phenomena and perhaps some work in mixed pure hydrogen and helium.

RADIATION AND PROPAGATION

01-1-46 THE RESONANT SLOT ANTENNA. J. Baker (Prof. Bevensee)

The radiation fields of one or more slots cut on the face of a rectangular waveguide are being studied. Each slot is about half a free space wavelength long at the operating frequency. By tapering the slot lengths and adjusting their spacing properly it is hoped to obtain a radiation pattern of high directivity and good bandwidth. This antenna has properties similar to a log periodic antenna.

21/02-1-18 STUDY OF FREQUENCY INDEPENDENT ANTENNAS. M. Gans (Prof. Rumsey)

Objective: To develop frequency-independent antennas which have any prescribed radiation pattern, polarization and impedance.

Operating tests had shown that the mode transducer, which is to be used to excite the planar equiangular spiral antenna in the n= 2, provides the desired output. A complete description of its construction and operation is given in: "Mode Transducer to Provide a Rotating TE_{21} Mode in a Hollow Circular Waveguide," M. S. Thesis, Michael J. Gans, University of California.

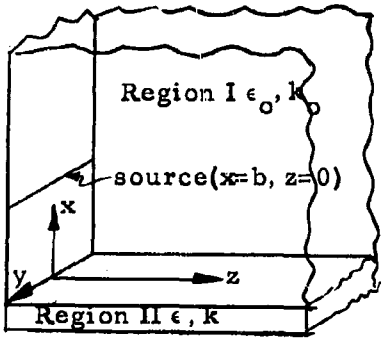
The experimental investigation of the propagation of waves over a plane sheet of sinusoidal wires is continuing. A thin sheet of teflon with parallel, closely spaced, sinusoidal strips photoengraved on one side is excited by placing one edge of it in a longitudinal slot in the broad face of a waveguide. However, since the sheets presently being used are not self-complementary surfaces, as assumed in the theoretical developments, self-complementary surfaces are being constructed for better correlation between the experiments and the theory.

Professor V. H. Rumsey has obtained the solution for the source free fields that can be supported by the sheet of sinusoidal wires. An attempt is being made to use these solutions as modes in which the field of a practical source located above the sheet could be expanded. It can be shown that these modes form a complete set by investigating the contour of integration which must be used to obtain the inverse transform of the transform of the field due to the practical source. We are presently trying to find the reciprocal basis for these modes. Alternatively, we are investigating methods by which losses can be introduced so as to make the modes become their own reciprocal basis and yet allow the important characteristics of the field in the loss-free case to be determined.

02-1-36 SURFACE WAVE INVESTIGATION. D. E. Norton (Prof. Rumsey)

A structure of the type shown below is known to support modes which are attenuated exponentially in the x direction in region I and propagate without loss in the z direction. The situation is complicated greatly, however, when consideration is given to exciting such a structure with a source which is finite in extent. In this case radiating as well as trapped waves are excited and the question as to whether the two types of waves exist separately and what is their nature has not been well resolved. For this reason an investigation of the structure shown below is being undertaken.

The waveguiding structure shown extends to infinity in the +x and +z directions, but is bounded by perfect conductors at $x = -t$, $y = \pm a$, and $z = 0$. The medium in region I is free space; in region II, a dielectric with dielectric constant ϵ. A narrow slot through the wall $z = 0$ at a height $x = b$ above the dielectric excites the structure. The field in the slot is $\vec{E}_s = \hat{a}_x E_o \cos(\frac{\pi}{2a} y)$.



For the purpose of calculation the electric field in the slot is replaced by a magnetic line current over the perfectly conducting plane. This current is

$$(1)\quad \vec{J}_m = -\hat{a}_y \frac{E_o}{2} \cos(\frac{\pi}{2a} y)\, \delta(x-b)\, \delta(z).$$

The fields in the structure may be expressed in terms of an electric vector potential, $\vec{F}$, which is related to $\vec{J}_m$ in the following manner:

$$(2)\quad \nabla^2 \vec{F} + k^2 \vec{F} = \epsilon \vec{J}_m.$$

An appropriate form for $\vec{F}$ with which it is possible to satisfy all the boundary conditions is

$$(3)\quad \vec{F} = \hat{a}_x f \sin(\frac{\pi}{2a} y) + \hat{a}_y g \cos(\frac{\pi}{2a} y).$$

The boundary conditions on f and g are

$$(4)\quad \frac{\partial f}{\partial z}\bigg|_{z=0} = \frac{\partial g}{\partial z}\bigg|_{z=0} = f\bigg|_{x=-t} = \frac{\partial g}{\partial x}\bigg|_{x=-t} = 0$$

$$(5)\quad \begin{cases} \frac{1}{\epsilon_o} f_1\Big|_{x=0} = \frac{1}{\epsilon} f_2\Big|_{x=0} \\ \frac{1}{\epsilon_o} \frac{\partial f_1}{\partial x}\Big|_{x=0} = \frac{1}{\epsilon} \frac{\partial f_2}{\partial x}\Big|_{x=0} + \frac{\pi}{2a} (\frac{1}{\epsilon_o} - \frac{1}{\epsilon}) g_1\Big|_{x=0} \end{cases}$$

$$(6)\begin{cases} g_1\Big|_{x=0} = g_2\Big|_{x=0} \\ \frac{1}{\epsilon_o}\frac{\partial g_1}{\partial x}\Big|_{x=0} = \frac{1}{\epsilon}\frac{\partial g_2}{\partial x}\Big|_{x=0} \end{cases}$$

Upon substituting (1) and (3) into (2) there results

$$(7)\begin{cases} \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial z^2} + [k^2(x) - (\frac{\pi}{2a})^2]f(x,z) = 0 \\ \frac{\partial^2 g}{\partial x^2} + \frac{\partial^2 g}{\partial z^2} + [k^2(x) - (\frac{\pi}{2a})^2]g = -\frac{\epsilon_o E_o}{2}\delta(x-b)\,\delta(z) \end{cases}$$

Thus the problem is to solve (7) for f and g subject to (4) through (6).

The future course of this project is to consider the solution of these equations as thoroughly as possible and to measure the field distribution in the structure experimentally.

01-1-42 FAR FIELDS FROM NEAR ZONE MEASUREMENTS. V. Galindo (Prof. Rumsey)

A method which will allow the direct measurement of the relative far field pattern of an antenna to be made in the near zone of the antenna is being investigated.

The method utilizes the following analytic expansion for any field component (in spherical coordinates), say H_ϕ;

$$H_\phi = \frac{e^{-jkr}}{r}\sum_{n=0}^{\infty}\frac{a_n(\theta,\phi)}{r^n},$$

which is valid for r taken exterior to a sphere containing the antenna (for an antenna of diameter D; $r > D/2$).

If N independent measurements of H_ϕ are taken, $H_{\phi_0}, \ldots H_{\phi_{N-1}}$, at distances from a prescribed origin of R_0, $R_0+\Delta r, \ldots, R_0+(N-1)\Delta r$, the following matrix relationship is developed:

$(H_{\phi_p}) = (L_{pn})(a_n)$ where $(L_{pn}) = f(\Delta, R_0) = N\text{x}N$.

It is possible to invert (L_{pn}) for arbitrary N with the result that

$$a_o(\theta,\phi) \propto \sum_{p=0}^{N-1}(-1)^p H_{\phi_p}\frac{(1+p\frac{\Delta r}{R_0})^N}{(N-1-p)!\,p!}e^{+jkp\Delta r}.$$

If the coefficients of H_{ϕ_p} are interpreted as the excitation coefficients of an array of elements at $R_0, \ldots, R+(N-1)\Delta r$, then the design of an array which will accomplish the objective has been obtained.

An upper bound for the remainder, after N terms, of the $\frac{1}{r}$ series for the Hertz potential, at a distance R_0, was found by using a Fresnel approximation for the potential as

$$< \frac{k^{N+\frac{1}{2}} \left(\frac{1}{R_0}\right)^{N+\frac{1}{2}} D^{2N+1}}{\sqrt{\pi}\ 2^{3N+1}(2N+1)\ N!} \qquad (k^2 = w^2\mu\epsilon = \left(\frac{2\pi}{\lambda}\right)^2).$$

For a $400\lambda = D$ antenna at $R_0 \approx \frac{1}{16}\left(\frac{2D^2}{\lambda}\right)$, this conservatively requires less than 20 terms for a negligible error.

It is necessary to investigate carefully the amplification of errors due to extrapolation; i.e., $(L_{pn})^{-1} H_{\phi_n}$ error. The effect of interactions must also be investigated with a view toward possibly using time sequential measurements of H_{ϕ_p} as opposed to simultaneously measuring H_{ϕ_p}.

02-1-11 ELECTROMAGNETIC SCATTERING FROM FERRITE CYLINDERS. C. Fong (Prof. Welch)

During the past quarter the fields scattered by cylindrical posts of two diameters, 3/4" and 7/8", were measured. As described in the previous report, the post is placed between parallel planes to simulate a cylinder of infinite length and is illuminated by a microwave field of 1.25" wavelength emanating from a horn placed between the planes. Patterns have been taken both with and without a D.C. magnetic field applied along the axis of the post.

For both post sizes, the patterns are symmetric about the horn axis when no D. C. field is applied. Application of the D. C. field shifts the main lobe to one side by as much as 15° for field strengths of 2000 Gauss. To check the results, the experiments were repeated with the post inverted, and the same results were observed. For the 3/4" post the main lobe intensity at 2300 Gauss was 4.5 db below the corresponding level at 1800 Gauss, showing resonance absorption. Similar results were observed with the 7/8" post, although the patterns were generally 5 db below those of the smaller post.

A complete report of all these measurements is currently under preparation.

SOLID STATE

01-1-56 MASER CRYSTAL STUDIES. H. Sloate (Prof. Bevensee)

A study has been initiated to determine the optimum orientation of maser crystals with respect to the magnetic field within a cavity, in order to obtain good power transfer from the crystal states to the cavity field. The orientation depends primarily upon the form of the spin Hamiltonian in the crystal.

01-1-57 TUNNEL DIODE HIGH-FREQUENCY OSCILLATOR. C. Ly (Prof. Bevensee)

Due to the difficulty of obtaining tunnel diodes with sufficiently high self resonant frequencies above 3 kmc, it has been decided to build an oscillator consisting of one or more diodes connected near the end of a coaxial waveguide. The oscillation frequency will be about 100 kc and variable by means of turning screws in the waveguide wall. This experiment will study the r. f. power saturation as a function of the number of diodes used and the discrepancies in their individual characteristics.

01-1-58 CRYSTAL AND MOLECULAR STUDIES. (Prof. Bevensee)

Objective: To obtain from the study of basic crystal physics a better understanding of transistor and maser phenomena.

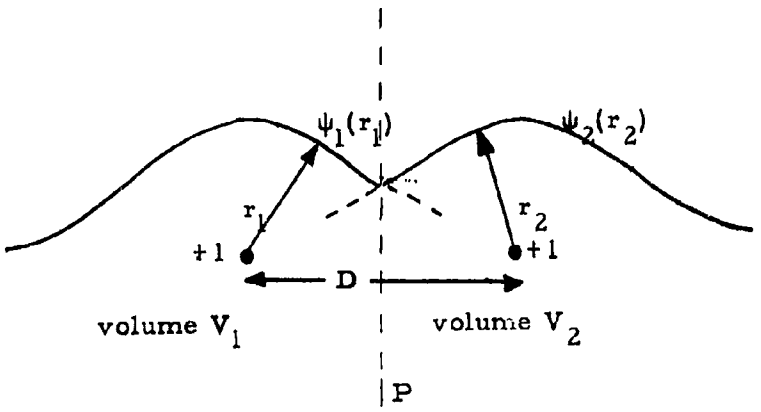
A general variational expression for the energy levels of a periodic crystal lattice has been presented by the writer. In order to verify the usefulness of this expression we used it to determine the first or lowest energy of the H_2 molecule. The variational expression can be modified so as to apply to this single molecule. We obtained a much better value for the energy than did many earlier workers who used more complicated expressions for the wavefunction. Since our calculation leads to an explanation for the stability of the molecule which does not involve exchange integrals we offer more details below.

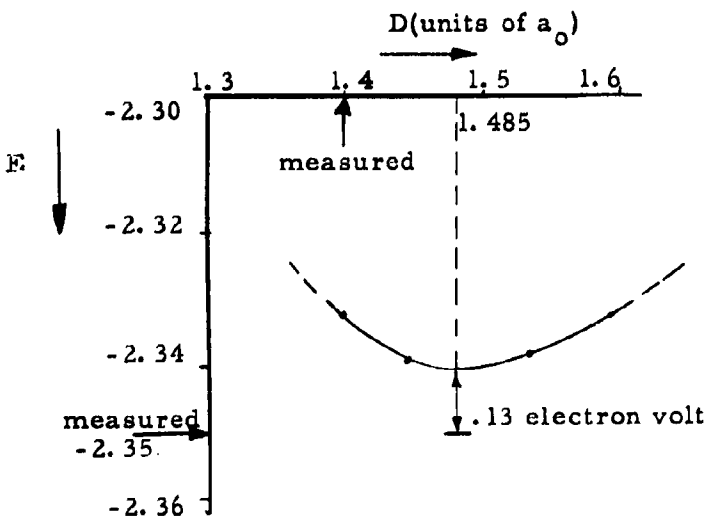
Shown here are two hydrogen nuclei or protons separated by distance D. Around each of them we assume the one-electron wavefunction varies as Ne^{-Zr/a_o}, $a_o = .529$ Å being the Bohr radius. N normalizes each wavefunction to unity:

$$\int_{V_1} |\psi_1(r_1)|^2 dV_1 = \int_{V_2} |\psi_2(r_2)|^2 dV_2 = 1.$$

Notice that each wavefunction is "chopped off" at the boundary plane P, so a discontinuity in gradient exists there; the wavefunctions, therefore, do not overlap. We account for this by a surface integral in the expression for energy E of this spacially-symmetric (and spin antisymmetric) state.

This wavefunction substituted into our variational expression for energy





E is in units of $\frac{e^2}{2a_o}$, one of which is 13.53 e.v.

E yields

$$\frac{1}{2}(E-E_1)X_1 = -\frac{2X_2}{DX_1+X_3} + \frac{X_1}{D} + \frac{X_2}{DX_1+2X_3} - \frac{1}{2}D\epsilon^{-ZD}$$

(electron-proton attraction) (proton repulsion) (electron repulsion) (discontinuity term)

with

$$X_1 = Z^{-3} - \epsilon^{-ZD}(\frac{1}{2}Z^{-3} + \frac{D}{4}Z^{-2}), \text{ the normalizer.}$$

$$X_2 = Z^{-6} - \epsilon^{-ZD}(Z^{-6} + \frac{1}{2}DZ^{-5}) + \epsilon^{-2ZD}(\frac{1}{4}Z^{-6} + \frac{1}{4}DZ^{-5} + \frac{1}{16}D^2Z^{-4})$$

$$X_3 = \epsilon^{-ZD}(\frac{3}{8}Z^{-4} + \frac{3}{8}DZ^{-3} + \frac{1}{8}D^2Z^{-2}).$$

E_1 is the energy of two isolated atoms and $(E-E_1)$ is the dissociation energy. This expression shows that E is not extremal with respect to variations in Z so we take Z = 1. The minimal energy is E = -2.34 units of $(e^2/2a_o)$, one unit of which is 13.53 e.v., for a D of 1.485, whereas the measured values are -2.35 and 1.40.

Our energy is high by only .13 e.v., whereas calculations of similar and greater complexity referred to by Pauling and Wilson[1] and by Seitz[2] are erroneous by about .5 e.v. We should note that these latter calculations obtain the value of D = 1.40 exactly. We believe that our relatively simple calculation with an error of only 3% in the dissociation energy and 6% in D offers an alternative explanation for the stability of the molecule. Instead of saying the exchange energy creates the energy minimum we could just as well attribute the minimum to large kinetic energy near the P-boundary (large negative $-(\hbar^2/2m)\nabla^2\psi$) where an electron leaves the influence of one nucleus and enters the region of influence of the other one.

Whether this interpretation of stability is more fundamental than the usual one remains to be seen, after more complex molecules are examined.

32-9-01 SILICON CARBIDE. R. E. Drews (Prof. English)

A final report on this work is essentially complete and ready for reproduction.

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[1] L. Pauling and E. B. Wilson, Introduction to Quantum Mechanics, McGraw Hill Co., 1935, page 340.

[2] F. Seitz, The Modern Theory of Solids, McGraw Hill Co., 1940, page 258.

32-9-02 CIRCUIT-INDEPENDENT OSCILLATIONS IN SOLIDS. S. Kakihana (Prof. English)

In the oscillistor phenomenon the frequency of oscillation, wave shape and the ratio of the ac to the dc component depend strongly on the relative orientation of the current flow line in the sample and the applied magnetic field. As has been reported by others the oscillation disappears when the angle between the magnetic field and the current flow line is made more than a certain critical angle (around 10 degrees). The modes of oscillation are quite different in general when the sample is rotated one way or the other with respect to the magnetic field.

Thus, in order to obtain meaningful data on the oscillistor phenomenon it is essential to align the crystal with the magnetic field and, for the study of the angular dependence of the phenomenon, to be able to vary the angle between the magnetic field and the current flow accurately. For this purpose a platform with rotating table supporting the dewar and sample was constructed. The sample will be aligned with the magnetic field by means of a bismuth-film Hall detector which is mounted on the sample holder.

02-1-26 ANOMALOUS PHOTOVOLTAIC EFFECT (APE). F. Junga (Prof. English)

Measurement of the spectral response of the APE on strained crystals of ZnS have been made. The crystals were stressed uniaxially along the hexagonal "c" axis. The photovoltage is also measured along the "c" axis. Stresses up to approximately 2800 psi have been applied to some crystals.

Stressing the crystal results in a shift in the wavelength at which the polarity of the photovoltage reverses. This shift cannot be accounted for in terms of a shift in band edge due to the applied stress. The only obvious explanation is that the applied stress alters the internal stresses present in the crystal. These internal stresses, according to A. R. Hutson, are responsible for the APE, through the piezoelectric properties of ZnS. This experiment adds weight to Hutson's model, but other models cannot as yet be ruled out. Therefore, other experiments aimed specifically at testing Tauc's model are also being performed. These experiments include studying the effect of added impurities on the spectral response of the APE.

32-1-02 MICRO-PLASMA BREAKDOWN EFFECTS. H. Powers (Prof. English)

A project has been initiated to study the phenomenon of microplasma breakdown in pn junctions with the ultimate purpose of obtaining a deeper insight into the various breakdown effects recently studied in the wide-band-gap semiconductors, silicon carbide and selenium. Since it is not known for certain whether these substances show uncomplicated plasma effects, the work will be initially oriented toward silicon devices.

02-1-15 MICROWAVE MODULATION OF LIGHT. G. C. Alexander (Prof. Singer)

Study has been devoted to three areas of the overall project.

First, several means of high-frequency modulating the microwave pumping power have been considered. At present, a voltage-sensitive diode switch with 10 manosecond switching time seems to offer the most economical means of modulation. A unit has been adopted into our first design.

Second, methods of growing ethylsulfate crystals doped with paramagnetic rare earth ions are being investigated. By varying the paramagnetic ion concentration, the relaxation time can be controlled. Cerium is seriously being considered as a short-relaxation-time ion with large Faraday rotatory power at liquid helium and hydrogen temperatures.

Third, study of the quantum-mechanical and classical models for Faraday rotation has been undertaken. The variation of rotation of the plane of light polarization when a microwave transition terminating the optical transition involved in the transition is most readily explained quantum-mechanically. The classical model based on the effect of electronic orbital motion does not easily permit an explanation of the effect of microwave pumping. The theoretical analysis is proceeding using a quantum mechanical formulation.

02-1-22 OPTICAL MASERS BY THE METHOD OF ELECTRON EXCITATION. L. Lin (Prof. Singer)

The purpose of this research project is the investigation of the possibility of achieving optical maser oscillations in a gas or vapor excited by inelastic collisions with incident electrons. In this scheme gas molecules in the ground level, upon being collided by electrons, make transitions to excited levels. Maser oscillation may result, between two excited levels, if certain conditions are satisfied.

An extensive preliminary theoretical analysis of this problem has been made. The results of this study are reported in Electronics Research Laboratory technical report Series No. 60, Issue No. 407.

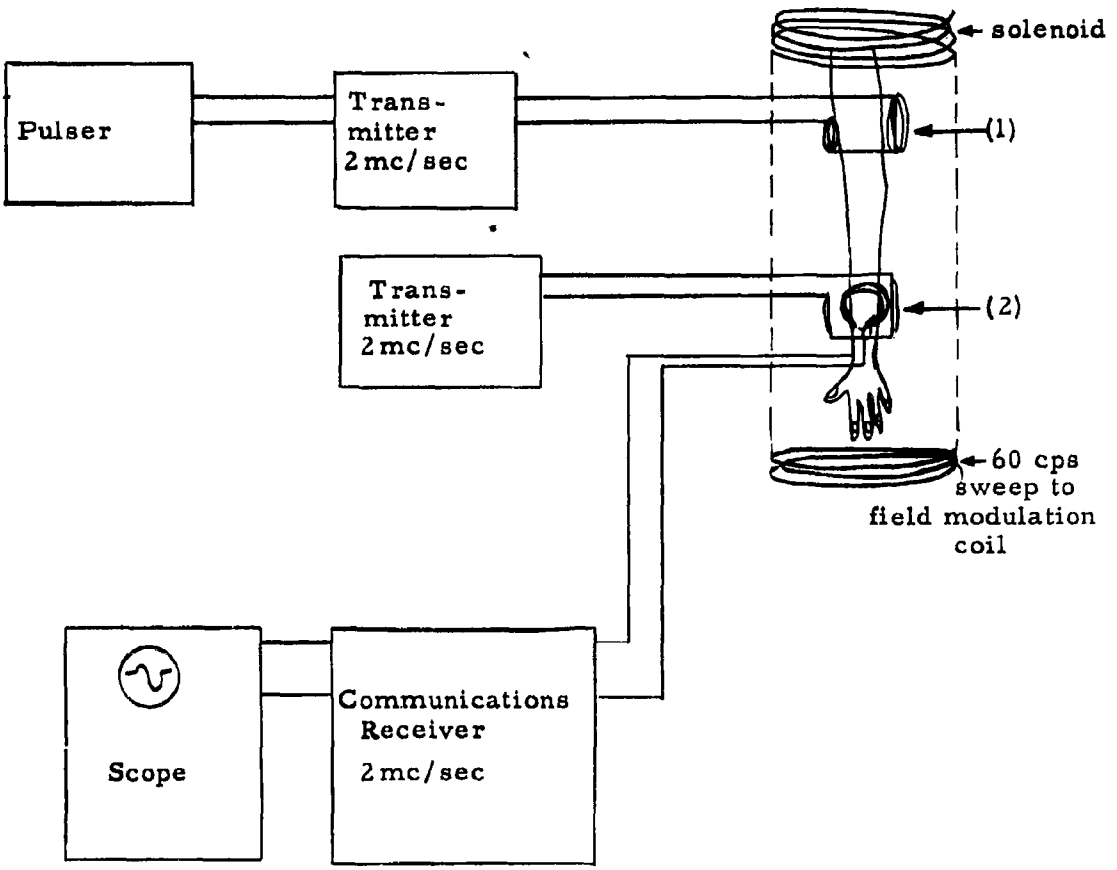
In particular, maser oscillation between 6^1P_1 and 7^1S_0 levels in mercury has been shown to be very promising.

At present efforts are concentrated on the design of a high pervian thermionic electron gun which will give an electron current of one ampere at approximately eight volts.

A high vacuum system for this purpose is under construction and is near completion. The first model of the electron gun has been constructed and will be tested in the high vacuum system.

02-1-35 NUCLEAR MAGNETIC RESONANCE BLOOD FLOW METER.
S. R. Levine (Prof. Singer)

Purpose: To measure the blood flow velocity in the human arm with no connections to the body. A rough picture of the apparatus is shown in the figure.



Nuclear resonance is observed with the second set of coils (2). The exact frequency of the transmitter is determined by the field strength according to the relation

$$f = \frac{2\mu H}{h} = \frac{\gamma H}{2\pi}$$

where
f = freq. required for resonance
h = planks const.
γ = magnetogyric ratio of the nucleus

For protons this becomes f = 4. 26H.

A burst of r. f. is sent through the first set of coils. This causes the protons to flip from the upper energy levels to the lower energy levels. With blood, this spin reversal lasts about 0. 4 sec. The blood which has been "tagged" can be detected by the second set of coils since this blood (the blood which has been "tagged") will absorb less energy from the r.f. field than the blood which has not been tagged. If we know the distance between the two sets of coils and the time between the pulsed r.f.and the detected signal, we can find the average velocity of the blood.

This theory (and initial experiments with blood flow in mice), has been set forth by Dr. J. R. Singer in a series of papers. The present task is to reduce theory to practice for human blood flow. The past 2 months have been spent in preparing the equipment for the experiment. One major concern has been in correcting the solenoid field with end correcting coils to produce a homogeneous field. A flux meter operating on the ball effect principle was constructed for measuring this field. The rest of the time was spent on observing Nuclear Magnetic Resonance in water doped with ferric chloride. This was done to determine the most efficient method of detecting N. M. R.

At the present time, we are concerned with the design of the coils to be used in the blood flow meter.

15-0-02 INFRARED AND OPTICAL MASERS. I. Gorog (Prof. Singer)

We are continuing the study of utilizing the excited states in alkali metal atoms, obtained as products of photodissociation of alkali-halide molecules for purposes of maser oscillation. In order to achieve continuous emission of coherent light from such a system, the employment of a molecular beam arrangement is desirable. The case of rubidium-iodide has been analyzed, considering the $Rb^F(5p\,^2P_{3/2} \rightarrow 5s\,^2S_{1/2})$ transition in detail. Our calculations show that operation of a beam-maser system, where the excited rubidium atoms are obtained by photodissociating rubidium-iodide is possible. However, in order to have an efficient system and a low noise output, the number of particles in the excited state should

be considerably greater than the number calculated from the minimum conditions of oscillation.

Further experimental and theoretical studies will be conducted to get a better understanding of the processes involved, especially that of photodissociation of diatomic molecules in molecular beams.

24-1-01 RbI OPTICAL MASER. C. W. Hudelson (Prof. Singer)

Investigations have been centered mainly in the area connected with optical pumping of RbI in the gaseous state. Of prime importance is the excitation of the particles with the proper ultra-violet wavelength, without allowing radiation of the same wavelength as that emitted by the sample to pass directly from the light source to the detector. E. g., a means must be found to allow wavelengths shorter than 2548 Angstroms to dissociate the particles, but prohibit 7800 Angstroms to pass through directly to the detector. Commercial filters, Christiansen filters and monochrometers are some of the possibilities being investigated. The major difficulty is that most reliable schemes only provide very low transmission at the wavelengths required.

Preliminary preparations are also under way on an experiment whereby a sample of RbI vapor is contained in a 2 inch quartz cube. The ultraviolet radiation is permitted to excite the particles and the intensity of 7800 Angstrom light emitted by the transition to the lower state is measured. This involves a specially designed high temperature furnace to contain the quartz cube, and a photo detection scheme, as well as the filtering problem mentioned above. Work is being continued in this direction.

Measurement of the spectral absorption coefficients of RbI vapor is also being considered as a possible experiment.

24-1-02 ELECTRIC DIPOLE MOMENTS. A. Pine (Prof. Singer)

We have undertaken to invert populations in a ruby optical maser by electric dipole transitions. The present state of the art in solid state laser pumping requires a three level system; the experiment done here will be to attempt to cause inversion using only the pertinent two levels in ruby - the R_1 line and the ground state complex.

The above method of pumping is known alternatively as 180° pulse inversion, coherent resonant pumping, and electric dipole inversion. It utilizes a strong, coherent radiation in the maser mode of oscillation at the frequency of oscillation. The Boltzmann population distribution shifts sinusoidally from ground state to excited state according to the Rabi strong field perturbation formulas. The angular frequency of this "modulation" of population is

$$\omega_1 = 2\langle R_1 | e\vec{x} | G \rangle \frac{e}{\hbar} \int_0^{\tau} E_x(t)\, dt$$

where $\langle R_1 | e\vec{x} | G \rangle$ is the electric dipole matrix element between R_1 and ground level.

$E_x(t)$ is the excitation field in the cavity.

τ is the length of time the excitation is applied.

If τ is adjusted so that $\omega_1 \tau = \frac{\pi}{2}$, then the populations will be left inverted. Maser oscillations of extremely high peak power and short duration may then result.

The system as planned will use a ruby optical maser as the excitation field source; this primary laser will be flash lamp excited. A Kerr cell shutter with speeds up to 5 manoseconds will be used to sample the primary laser output. Synchronization and detection techniques are presently the concern of the experimenters.

02-1-14 STUDY OF SEMICONDUCTOR PHENOMENA UNDER HIGH-FIELD CONDITIONS. H. Plutchok and S. Tee (Prof. Wang)

Work has been initiated to study the tunneling mechanism through thin insulating films. Preliminary experimental results on Al_2O_3 film show that the current increases linearly with the applied voltage for very low voltages and then increases very rapidly and nonlinearly with the applied voltage around 200 mv. So far, the results are not yet reproducible mainly because it is hard to get films of uniform and consistent thickness.

Experimental investigation on measuring semiconductor properties at microwave frequency is also under way. The method is intended for studying hot electronic effects in semiconductors under high-field conditions.

02-1-20 STUDY OF NONLINEAR BEHAVIOR OF GARNET AT HIGH MICROWAVE POWERS. G. Bodway and G. Kemanis (Prof. Wang)

Restabilization of an unstable spin system by means of modulating the d. c. magnetic field is being studied. Recent analytical work by Suhl on square-wave modulation does not seem to corroborate experimental results by Weiss, regarding the dependence of the modulation field strength upon the frequency of the modulation field. An attempt is currently being made to analyze the sinusoidal modulation case and to obtain an approximate analytical solution between the growth constant, modulation amplitude and modulation frequency.

Experimentally, the electromagnet with its regulated power supply is now in good operating condition. The microwave equipment is being assembled. The current effort is in building a resonant cavity that has a wall thin enough to let in the modulating field and still has a reasonable Q at microwave frequency. A tuned amplifier and a coil fitted to the outside of the cavity are being constructed. It is expected that the assembly will produce a modulation field of few gauss at 1 mc in the cavity.

SYSTEMS

41-1-06 OPTIMAL CONTROL AND THE CALCULUS OF VARIATIONS. A. Larsen (Prof. Bergen)

To obtain a more thorough understanding of Pontriagin's maximum principle an investigation of alternate derivations of the principle has been undertaken. Of considerable interest is the use of the classical calculus of variations suitably modified to account for inequality constraints. Since in this case it appears that the maximum principle may be obtained in the form of the classical Weirstrauss E-function condition it is of the greatest interest to compare the assumptions pertaining to the two derivations.

43-1-04 STABILITY OF NONLINEAR CONTROL SYSTEMS. I. J. Williams (Prof. Bergen)

A technical report which proves the validity of Aizerman's conjecture for any control system having three poles and one nonlinear gain function is in preparation. A technical report specifying the types of control systems for which Aizerman's conjecture has been validated is also in preparation.

Lyapunov functions have been determined which verify Aizerman's conjecture for some types of second order systems having two zeros and two poles and for third order systems having a zero at the origin and three poles. This latter result generalizes the case of the Russian author E.A. Barbashin who verified the conjecture for systems having one zero at the origin and three poles which are the roots of the polynomial, $s^3 + as + b$.

The Lyapunov functions used for these two cases were determined directly by considering a particular physical system whose dynamic behavior is governed by the same set of differential equations as that of the system in question, and whose energy dissipation is a function of the output variable. The energy stored in the physical system is then used as a Lyapunov function to obtain the results stated. In these two cases the nonlinear element was identified as a nonlinear dissipation element. In other instances the nonlinear element can be identified as a nonlinear energy storage element.

30-1-02 SEQUENTIAL DECISION PROCESS WITH DISCOUNTING FOR A FINITE STATE PROBABILISTIC MACHINE. (Prof. Blackwell)

This project is completed and a manuscript containing the results has been submitted for publication. It has been shown that there always exists a stationary policy which is optimum for all discount factors close to one. Methods for finding such optimum, and nearly optimum, policies are developed in detail.

02-1-16 STUDY OF NONLINEAR SAMPLED-DATA CONTROL SYSTEMS. S. Kodama (Prof. Desoer)

An explicit formula giving the stability bounds of gain variation for a class of sampled-data control systems described by $\underline{x}_{k+1} = \underline{A}\,\underline{x}_k + \underline{h}\,g(x_i(k))\,x_i(k)$ has been derived using a quadratic Lyapunov function. ($\underline{x}_k$ and $x_i(k)$ denote the state vector and its ith component both at the kth sampling instant, $\underline{A}$ and $\underline{h}$ are nxn and nxl constant matrices and $g(x_i)$ is a nonlinear-gain parameter.) The formula specifies a gain sector (g_2, g_1) in terms of $\underline{A}$, $\underline{h}$ and g_o, where g_o is a fixed gain such that the linear system $\underline{x}_{k+1} = \underline{A}\,\underline{x}_k + \underline{h}\,g_o\,x_i(k)$ is asymptotically stable. If $g(x_i)$ is such that $g_2 < g(x_i) < g_1$ for all x_i, then the nonlinear system is assured to be asymptotically stable in the large. Finally, a method to determine the coefficients of the quadratic Lyapunov function to maximize the gain-sector (g_2, g_1) is given. A numerical study with a second-order nonlinear servo has shown that the above-mentioned procedure yields a sharp stability criterion.

10-1-01 THE MINIMAL TIME PROBLEM FOR LINEAR DISCRETE SYSTEMS. J. Wing (Prof. Desoer)

This work considers a linear time invariant discrete system whose state transition equation is given by

$$\underline{x}_{k+1} = \underline{A}\,\underline{x}_k + u_{k+1}\,\underline{a}$$

where $\underline{A}$ = nxn constant matrix, $\underline{x}_k$ is an n-rowed state vector of the system at $t=kT$, and $\underline{a}$ is an n-rowed control vector. The control u_{k+1} is restricted to be an admissible control, i.e., $|u_{k+1}| \leq 1$ for $k=1, 2, \ldots$. It is assumed that S is completely controllable by admissible control. The minimal time problem may be stated as follows:

i) Given any arbitrary initial state $\underline{x}_o$ and any arbitrary target state $\underline{x}_t$ of the system, find the admissible control $u_1, u_2, \ldots$ which will transfer $\underline{x}_o$ to $\underline{x}_t$ in the minimum number of sampling periods.

ii) Determine an optimal strategy, i.e., determine a scalar valued function $u^o(\underline{x}_o, \underline{x}_t)$ such that if the system is at $\underline{x}_o$ and $\underline{x}_t$ is the target state at the sampling instant, then $u^o(\underline{x}_o, \underline{x}_t)$ is an optimal control for the next sampling period.

Unlike the minimal time regulator problem, i.e., $\underline{x}_t = 0$ for which the existence of $u^o(\underline{x}_o, 0)$ for any $\underline{x}_o$ is guaranteed by controllability, the minimal time problem in general does not have $u^o(\underline{x}_o, \underline{x}_t)$ for arbitrary $\underline{x}_o$, $\underline{x}_t$. By defining a special set $\Omega(0)$ in the state space and limiting $\underline{x}_t \in \Omega(0)$ the existence of $u^o(\underline{x}_o, \underline{x}_t)$ is guaranteed for any $\underline{x}_o$ in the state space.

The problem of determining $u^o(\underline{x}_o, \underline{x}_t)$ for S, if $u^o(\underline{x}_o, \underline{x}_t)$ exists, can be transformed to be a problem of minimal time regulator for S provided N, the miminum number of sampling periods required to transfer $\underline{x}_o$ to $\underline{x}_t$ by admissible controls, is known. The determination of N is tractable only in very special cases. Nonetheless, it is true that $u^o(\underline{x}_o, \underline{x}_t) = u^o(\underline{x}_o - \underline{A}^{-N}\underline{x}_t, \underline{0})$.

02-1-25 ENERGY OPTIMAL CONTROL. C. Lee (Prof. Desoer)

The project was inactive during this quarter due to the absence of the staff during this time.

02-1-21 ITERATIVE TECHNIQUES FOR GENERATING THE OPTIMAL CONTROL AS A FUNCTION OF THE STATE. (Prof. Desoer)

The problem is still in its preliminary stages and there are no results worth reporting so far.

02-1-30 THE SOLUTION OF LINEAR PROBLEMS BY USE OF THE ADJOINT SYSTEM. R. Sussman (Prof. Desoer)

Every system B, can be characterized by an operator L, which maps the input u of a system into its output v, i.e.,

$$v = L\,u$$

To every system B we can associate an adjoint system B* with input u*, output v* and operator L*, which is defined so as to satisfy the following relation:

$$\langle L\,u;\, u^* \rangle = \langle u;\, L^*\,u^* \rangle \qquad (1)$$

for a specific definition of the inner product.

From (1) we immediately get the following Reciprocity Theorem:

$$\langle v;\, u^* \rangle = \langle u;\, v^* \rangle \qquad (2)$$

and as a consequence of (2) we get (3)

$$v = \langle u;\, G^* \rangle \qquad (3)$$

where G* is the "impulsive response" of the adjoint system B*.

The close interdependence between the system and its adjoint is clearly exhibited by relations (2) and (3).

By use of these relations several specific problems in systems and in circuits have been worked out. It is hoped that by a systematic application of the above-mentioned results, it will be possible to develop a general procedure of solving a certain class of problems, by use of the adjoint problem.

02-1-34 SIMULATION OF FINITE-STATE AUTOMATA. M. Silverstein (Prof. Gill)

Statement of the problem: A method is sought whereby finite state automata may be simulated in a digital computer. Included in the simulation are a set of commands which will perform operations on the automata, such as minimization of the number of states, identification of initial state, passing to a known state and others.

Approach to the problem: An automata is represented as a device with a finite number of binary inputs, a finite number of binary outputs and a finite number of states. The dynamics of the automata are completely specified by Boolean expressions which, for each state, determine the outputs and next state as a function of the inputs. Algorithms for manipulating these expressions in order to execute the above-mentioned operations must be found. The expressions are stored in a computer memory through the use of a list processing language, and subroutines are developed out of which the algorithms are built as routines. Major routines, or programs, will be developed which will accomplish various of the operations and/or combinations thereof.

Results to date: A general method for computer handling of Boolean algebra has been developed. It may be programmed in any language employing the notion of lists or associative memory; IPL-V is being used to obtain specific results. Commands exist for Boolean addition, multiplication and complementation of expressions. A small subroutine will make logical reductions. An algorithm has been found for the minimization of the automata, i.e., detection and elimination of equivalent states. Given a set of Boolean expressions representing a machine, the algorithm will determine a new set of expressions representing an equivalent minimal machine. This algorithm is presently in the process of being programmed in the command structure developed for computer manipulation of Boolean expressions.

01-1-48 A STUDY OF THE PARALLEL ACCESS INFORMATION RETRIEVAL SYSTEM. D. Stone (Prof. Gill)

Motivation for the study: Research workers have been denied penetrating and comprehensive access to their own graphic records by the large and growing size of the collection of those records. Attempts to provide such access have been given names such as: Information Storage and Retrieval, Data Banking and Selective Searching. These attempts have made use of existing machinery ranging in complexity from digital computers to edge-notched cards. But the IR problem involves information which is stored in large quantities, is invariant in the store and is not subject to high-speed numerical calculations. A solution to this problem which takes advantage of these characteristics is outlined below.

Purpose of the study: The purpose of the study is to devise an economically feasible information retrieval machine of minimum complexity and maximum effectiveness, suitable for growing information stores of widely different sizes.

Method: a) Become familiar with the literature on IR, so as to be able to understand and define problems fundamental to the IR process.

b) Investigate the logical operations possible in a static store and decide on the best compromise between effectiveness and complexity.

c) Examine physical means of realizing the IR machine.

Results: Part a) of the method has been done, and a decision has been made to confine attention to the parallel access information retrieval system.

01-1-49 IDENTIFIER PREPROCESSOR FOR ALGOL-TYPE LANGUAGES. G. Anderson (Prof. Huskey)

The first stage device of a generalized ALGOL compiler has been programmed. The primary function of this unit is to relieve the translating routine of the compiler of the duty of memory allocation. By doing this it is possible to use a translator of higher speed and lower complexity, and the final assembly stage is much simpler.

The preprocessor has been programmed in the NELIAC language on the IBM 704 and the present effort has been directed toward making the unit operational as well as coordinating the details of the system with the details of the two other main components of the ALGOL compiler.

01-1-50 AUTOMATIC DECLARATION. J. Spitze (Prof. Huskey)

The processing of declaration statements (of ALGOL or NELIAC type) has been studied. An "Analyser" is now being written in NELIAC which will scan a NELIAC- or ALGOL-type program and generate the proper declaration statements. It will still be necessary to "declare" the basic form of arithmetic, i.e., floating point or integer. Indices will be automatically identified as integer variables and the appropriate declarations will be generated. Similarly, non-dynamic arrays will be identified and multiple subscripts will be flagged so that the subsequent stages of the translator can generate efficient coding.

01-1-51 AN ALGORITHM FOR THE TRANSLATION OF ALGOL. W. M. Keese (Prof. Huskey)

An algorithm for the translation of an ALGOL-type language into a machine independent intermediate language has been developed. The source language may include all algebraic (arithmetic and Boolean)

statements of AGLOL and/or NELIAC, ALGOL conditional statements, transfers and labelings, as well as more general algebraic statements than are permissible to neither ALGOL nor NELIAC.

Currently this translator is being adjusted for compatibility with a preprocessor and an intermediate language assembler.

In the future there are plans to develop a better intermediate language (a specially structured generalization of the Polish string) which seems as if it should prove more general, easier to assemble and more easy to translate intc. There are also plans to:

1) Extension of problem oriented languages to the inclusion of more mathematical usages than they now enjoy, particularly those of set, quantification and more general operator, relation, function (either built-in or establishable).

2) Incorporation and/or establishability of metalinguistic capabilities (including data manipulation) both in themselves and as a special case of 10.

01-1-52 INTERMEDIATE LANGUAGE ASSEMBLY PROGRAM.
B.G. Middleditch (Prof. Huskey)

An assembly program is being programmed to read a Machine Independent "Intermediate Language" and assemble a program for the 704. A limited version is now running on the 704. Functions, subroutine transfers and FOR statements are being checked out.

It is planned to have a minimum or bootstrapping assembly program written in the Intermediate Language which may be translated into machine code for other machines and which in turn can assemble the main assembly program for the other machine. This will depend on being able to express the main assembly program in terms of the Intermediate Language.

01-1-53 OPTIMIZATION OF MULTIPLE OUTPUT BOOLEAN FUNCTIONS.
P. White (Prof. Huskey)

A procedure for the optimization of multiple output functions has been developed. This is an algorithm for the determination of the most economical p-input, q-output switching circuit required for the implementation of q Boolean switching functions of p variables each.

The proofs for the procedure have just been completed. A program for the automation of the procedure has alsc just been completed and will be run on the IBM 709 at the Lawrence Radiation Laboratory. The program is written in the NELIAC computer language, and is capable of processing problems of up to ten inputs and fifteen outputs.

02-1-31 A NELIAC COMPILER FOR IPL-V. R. Love (Prof. Huskey)

Research is being conducted on developing a program to compile Information Processing Language V routines. IPL-V is a symbol and list-structure manipulating language presently implemented as an interpretive system. To study the operation of the interpreter a Symbolic Assembly Program listing of IPL-V has been obtained. From this listing the interpreter has been rewritten in the NELIAC language. This transformation allows one to more easily visualize the operation of the IPL-V system. Tests are presently being run on the NELIAC program to insure its accuracy. The knowledge from this NELIAC version of the IPL-V interpreter will be used to construct a compiler of IPL-V. To build the compiler three major steps will be performed: 1) development of an indexable list; 2) program to compile instructions to perform the operations of the interpreter; and 3) writing basic processes to manipulate the lists and symbols. Once the IPL-V compiler is written it may be combined with the NELIAC compiler to allow efficient algebraic operations during list processing.

02-1-32 AUTOMATION OF LITERATURE ABSTRACTING. J. White (Prof. Huskey)

The primary aim of the project is the development of a literary and scientific reviewing algorithm which will carry out as its chief duty the automatic creation of literature abstracts.

The first part of the period was devoted to the study of the now existing methods of abstracting, such as that of H. Luhn. These proved to be unsatisfactory mainly because of their inconsistency in turning out acceptable abstracts. For this reason research turned towards a more suitable approach to the problem.

During recent years the field of structural linguistics has been advancing rapidly. New techniques have been developed towards a rigorous and scientific approach to language, making the problems of syntactic and semantic classification much easier. The main emphasis of this project, therefore, has been on the acquiring of a working knowledge of this linguistic theory. To this end extensive literature search, course work in linguistics and frequent discussions with men in the field of structural linguistics have been undertaken.

Even at this early stage it is believed an algorithm using these methods will make possible not only acceptable but quite suitable abstracts for articles in particular fields of research.

02-1-33 SIMPLIFICATION OF BOOLEAN FUNCTIONS USING MATHEMATICAL PROGRAMMED TECHNIQUES. M. A. Breuer (Prof. Huskey)

Techniques in Linear (Mathematical) programming have been applied to Boolean functions by Prather and by Schaeffer. Integer programming methods are being investigated and the technique is being extended to non-linear objective functions.

10-0-02 OSCILLATIONS IN NONLINEAR SAMPLED-DATA SYSTEMS. M.A. Pai (Prof. Jury)

A paper has been submitted for publication in AIEE Transactions. In this paper a method is presented to examine the necessary conditions for the existence of certain limit cycles in nonlinear sampled-data systems. The method is based on the principle of specifying a certain repetitive output from the nonlinearity and finding under what conditions this sequence will be sustained. Many examples are worked to illustrate the method.

Also a particular example of forced oscillation is considered and an analytical treatment is presented for investigating the existence and stability of such forced oscillations.

10-1-02 TIME-VARYING SAMPLED-DATA SYSTEMS. C.A. Galtieri (Prof. Jury)

By a very simple extension of some of the methods used for the identification of linear, time-invariant sampled-data systems, a generalized approach has been developed. This applies to the identification of systems with known structure, i.e., systems whose mathematical model is assumed known a priori except for the numerical value of a finite number of constant parameters. It can be seen that for a large class of models the problem is reduced to the inversion of a matrix.

Research is now being conducted in the field of identification of the model structure. A statistical approach is being presently investigated.

10-1-05 NONLINEAR SEQUENTIAL CIRCUITS. A. Chang (Prof. Jury)

An investigation of linear and nonlinear sequential modular circuits was begun. The problem of determining the cyclic behavior on nonlinear, finite-state, autonomous systems was attacked. Bounds on the length of the noncyclic part of the behavior of such systems have been established in terms of the range of the nonlinear operator associated with the system. The results of this study will be reported in the Electronics Research Laboratory technical report entitled "Notes on System Theory, II."

02-1-24 STATISTICAL STUDY OF PULSE-WIDTH MODULATED SYSTEMS. S.C. Gupta (Prof. Jury)

Computer solutions for closed loop PWM control systems have been obtained by use of the IBM 704. These are the numerical solutions of the implicit equations involving the MSV of output and the MSV of input. Signal plus noise has also been considered for the open loop case giving non-stationary output. It has been found that discrete compensation is better to have the MSV of output equal to the MSV of input. Some simple examples have been considered in this synthesis. Some experimental work using the IBM 704 is under way to examine the nature of errors involved in approximations which have been used in the analysis and synthesis of PWM control systems.

43-1-16 ON AN OPTIMAL DISCRETE SYSTEM. K. Sakuda (Prof. Polak)

An optimal strategy for time-optimal control of PWM systems having transfer function of the form $\frac{K}{(s - \lambda_1)(s - \lambda_2)}$, $\lambda_1 < \lambda_2 \leq 0$, was given by E. Polak[1] who also showed that this optimal strategy is not unique. The purpose of this project is twofold. First, it is proposed to find a time-optimal control strategy for the systems above such that the total control-pulse area is a minimum for each time-optimal transient process. In a gas ejection type servo this is equivalent to minimizing the total gas volume expended per minimal-time transient. Secondly, it is proposed to find a control strategy which would result in a minimal total control-pulse area for transient processes in which the duration is preassigned from outside considerations. The second problem arises from the fact that if minimal time is not essential, the total control-pulse area can be substantially reduced.

01-1-21 OPTIMIZING CONTROLS. R.T. Lacoss (Prof. Smith)

A digital computer study is being formulated for a dead-time system with load disturbances and a hypothesized optimum minor loop transference.

01-1-22 MAGNETIC DELAY LINES. C.E. Stoffers (Prof. Smith)

A magnetic delay line with compensation for accumulative bias, due to diode voltage drops, has been designed.

01-1-23 TURBO GENERATOR ROTOR ANGLE CONTROL. P.P. Varaija (Prof. Smith)

A frequency discriminator and torque angle indicator is being built to operate from an a-c generator mounted on the shaft of the main turbine.

01-1-59 MAXIMUM EFFORT CONTROLLERS. H.K. Knudsen (Prof. Smith)

An iterative technique of finding the correct initial conditions for the adjoint solution of a maximum effort system for an optimal trajectory has been developed.

01-1-60 PULSE RESPONSE OF NONDISTORTIONLESS LINES. S. Fine (Prof. Smith)

The pulse response of nondistortionless lines with reactive loads is being investigated.

01-1-61 OSCILLATION SUPPRESSOR FOR NON-REGULATED SUPPLY FOR PULSER. E.L. Harris (Prof. Smith)

A method of preventing power supply voltage oscillations through pulsing the screen grids of the transmitter tubes has been developed.

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[1] Polak, E., "Minimum Time Control of Second Order Pulse-Width-Modulated Sampled-Data Systems," ASME Paper JAC-7, JACC, June 1961, (ASME Journal of Basic Engineering, in press.)

01-0-24 HIGH RESOLUTION TV SYSTEM FOR STATIONARY PATTERNS. (Mr. Studer)

The purpose of this project is to establish criteria for correlating TV picture detail of stationary patterns and required resolution of TV systems.

Comparative tests will be carried out with high resolution and standard resolution systems, using the same camera tube and optics for both cases.

30-1-03 THE STRONG CONVERSE OF THE CODING THEOREM FOR CONTINUOUS CHANNELS. (Prof. Thomasian)

The purpose of this project is to find methods of proof which will apply in this more general context and yield computable bounds. No significant results have been obtained so far.

30-1-09 VARIABLE LENGTH ENCODING OF STOCHASTIC SYMBOLS. F.L. Morse (Prof. Thomasian)

The information rate is known for the variable length encoded version of equally likely independent symbols. The purpose of this project is to find the information rate when the symbols are not equally likely.

30-1-10 COUNTABLE STATE SEQUENTIAL MACHINES. G.K. Machol (Prof. Thomasian)

Both deterministic and stochastic sequential machines with a countable number of states are being examined. It is expected that such machines will be more realistic models of many control situations. It is not yet clear whether or not there are interesting generalizations of the known results for finite state machines.

30-1-11 BINARY DECODING. T.J. Wagner (Prof. Thomasian)

The problem is to find methods of decoding, for the binary symmetric channel, which require very little computation and are easily implemented. It has been shown that linear methods will not work. The literature on sequential decoding is being studied.

30-1-12 IMPROVED HAMMING CODES. R.N. Miller (Prof. Thomasian)

This project is essentially completed and is being written up. An explicit method of code construction was found which in every case either agrees with the corresponding Hamming code or improves it for error detection.

01-1-54 ANALYSIS OF TRACKING CONTROL SYSTEM WITH NOISE DISTURBANCES. R.E. Brooks (Prof. Turin)

A study is being made of a class of (nonlinear) tracking control systems with noise disturbances. The emphasis has been on a first-order system, for which explicit expressions for the probability of unlocking have been obtained. The results obtained are for systems tracking constant-velocity targets and having certain initial distributions of tracking error. Present work is concerned with the extension to higher-order systems and the generalization of initial conditions.

12-1-06 INFORMATION THEORETIC STUDY OF SYNCHRONIZATION. H. Kaneko (Prof. Turin)

The probability of error of a communication system with an optimum receiver is being studied when the synchronization is not correct. The average probability of error considering incorrect synchronization can be obtained by integrating the product of the probability of error for given sampling time and the a posteriori probability distribution of sampling time over the entire range of sampling time.

We have considered a case in which a sinusoidal wave is transmitted as a synchronizing signal and a sequence of binary information pulses is transmitted with the same period as the sinusoidal wave. The measurement of sampling time is made by finding the phase of the sinusoidal wave by an ideal phase detector. In this case the a posteriori distribution of the sampling time is the same as the distribution of phase of narrow band additive gaussian noise plus sinusoidal wave.

The average probability of error is computed for various values of parameters by the IBM 704 computer and the minimization of the average probability of error for a given sum of information signal power and synchronizing signal power is now being prepared for machine computation.

02-1-29 INVESTIGATION OF THE PROPERTIES OF THE ESTIMATE OF THE IMPULSIVE RESPONSE OF A LINEAR, TIME-INVARIANT SYSTEM IN THE PRESENCE OF NOISE. H. Kwakernaak (Prof. Turin)

This project has been completed and a report in the form of a Master's thesis is being prepared.

30-1-07 ADAPTIVE FILTER. H. Scudder (Prof. Turin)

Work has been started on the establishment of an exact mathematical definition of an adaptive filter, starting from a definition of perfect adaptivity given by Prof. Zadeh. The definition is as follows: A filter is designed according to a certain criterion which is based on the information available to the filter at time t_0, and the filter operates during the period from t_0 to a later time t_1. The filter is called perfectly adaptive if it changes its characteristics so that at time t_1 it is identical in operation to a filter which is designed using the same criterion based on the information available at time t_1.

Work is beginning on an example of asymptotic adaptivity. This example involves the analysis and design of an adaptive Wiener filter, which continually estimates the spectrum of its input, and uses this estimate to change its response to the input, so that the filter will asymptotically ($t_1 \rightarrow \infty$) approach the optimum linear filter.

02-1-27 IMPLEMENTATION OF FUNCTION STATEMENTS IN COMPILERS. N.E. Wirth (Prof. Wattenburg)

A routine which serves to translate arithmetic function statements from the NELIAC language into a sequence of machine instructions has been implemented in the NELIAC-MONITOR system. The necessary modifications are also under way to permit use of this routine in the NELIAC II system for the IBM 7090 computer.

51-1-01 AUTOMATIC PROGRAMMING SYSTEMS FOR LARGE-SCALE DIGITAL COMPUTERS. C. Conn (Prof. Wattenburg)

The 704 NELIAC system has been completed and is now being used in programming systems studies. The 704 NELIAC system is now being expanded for operation on the IBM 7090 computer. The increased speed and input-output facilities of the 7090 are being utilized in the 7090 NELIAC system.

Several other investigators are now using the NELIAC system in their research.

01-1-47 TUNNEL DIODE CIRCUITS. J.A. Lukes (Prof. Wattenburg)

In the immediate future an experimental four flip-flop counter employing a weighted binary code will be built. In this way the analysis presented by the literature may be coupled with the experience gained in the laboratory in order to gain insight into the inherent problems of the tunnel diode as a physical device. With the knowledge gained from the literature and laboratory experience a high speed arithmetic unit may then be designed and tested.

01-1-55 COMPUTER SYSTEMS DESIGN. A. Miyoshi (Prof. Wattenburg)

Various aspects of computer logical organization are being studied. The study involves a review of past and present digital computer techniques, in preparation for the design and specification of a simple electronic digital computer.

Sub-topics which are currently being studied include: 1) binary and decimal notation; 2) binary arithmetic; 3) address and address modification; 4) instruction sets; 5) microprogramming and macroprogramming; 6) input-output control; 7) Multiplexing and A-D conversion; 8) memories; and 9) synchronous and asynchronous organizations.

The major effort is centered on the development of a highly compatible system, utilizing standard circuitry in a unique system design.

At present several techniques seem worthy of further consideration for possible inclusion in the final design: 1) time-sharing of the arithmetic and control registers; 2) functional bit instruction coding; and 3) optimum programming. Two memory systems are presently under consideration: 1) a modified magnetic-tape system utilizing an endless magnetic-tape loop; and 2) a quasi random-access, TapeDrum system.

41-1-04 DETERMINISTIC PROCESSES AND PREDICTION OPERATORS. J.F.A. Ormsby (Prof. Zadeh)

Work is continuing on the problem of the determination of perfect predictors for certain stationary and non-stationary processes. Two papers dealing with the results to date are being readied for issue as Electronics Research Laboratory reports. Extensions of the analysis to discrete processes over finite fields as well as for non-stationary processes are being considered.

41-1-05 A CANCELLATION TECHNIQUE FOR TIME OPTIMAL CONTROL PROBLEMS. J.H. Eaton (Prof. Zadeh)

Work is continuing on the problem of taking a given system from a specified initial state to a specified terminal state in minimum time. For systems with amplitude constraints on the input it has previously been shown that the desired terminal state can be reached in minimum time using a bang-bang input, that is, an input which is always on the boundary of the space of constraints. This investigation is concerned with using the properties of the Laplace transform to find the required switching times for optimal control of linear systems. Using pole cancellation techniques a set of transcendental equations whose solutions yield the required switching times has been obtained for both the regulator problem, in which the desired terminal state is zero, and the tracking problem, in which the desired terminal state can be non-zero and can be time dependent. At present this investigation is centered around determining the relation between incremental changes of the initial state of the system and changes in the optimal switching times. The results obtained indicate that it may be feasible to use the optimal switching times for a given initial state to find the optimal switching times for an arbitrary initial state.

43-1-03 IDENTIFICATION PROBLEMS IN FINITE-STATE MACHINES. J. Raviv (Prof. Zadeh)

Research is being continued on the problem of taking a deterministic finite-state machine to a specified state in the shortest (expected) time, when one is given an initial probability distribution on the states.

At each stage the experimenter has to make a decision on what input to apply to the machine on the basis of an observed information pattern. After the input is applied and the output of the machine observed, the a posteriori probability distribution on the state space is calculated and the next decision is made on the basis of this distribution. The first problem which arises is of finding an optimal policy for taking the finite-state machine to a known terminal state without knowing the initial state. For purposes of decision making the probability distribution space is divided into connected regions by hypersurfaces such that a particular input is applied when the a posteriori probability distribution falls into a particular region. At present research is centered on the investigation of the forms of these hypersurfaces.

43-1-09 SYSTEM IDENTIFICATION. B. Whalen (Prof. Zadeh)

Work is continuing on means of identifying systems from their time domain input-output relationships. The effects of measurement errors are being studied.

43-1-10 DISCRETE-TIME INTERRUPTED STOCHASTIC CONTROL PROCESSES. J.H. Eaton (Prof. Zadeh)

This investigation is concerned with discrete-time processes that are interrupted in the sense that at each time t, $t = 1, 2, \ldots$ there is a probability p_t (called the probability of interruption) that the state of the system to be controlled cannot be observed. The radar tracking problem is a typical physical process which might be formulated in these terms. It has been shown that an optimal policy for controlling such interrupted processes can be obtained using the usual iterative techniques of dynamic programming. An interrupted process involving a linear system and a quadratic cost criterion has been examined in detail and an analytic expression for the optimal policy has been obtained. In this case the optimal policy is found to be independent of the probability of interruption. The results of this investigation are being extended to include continuous-time processes.

43-1-11 EQUIVALENCE IN LINEAR SYSTEMS. G. Bacon (Prof. Zadeh)

The study of equivalence in rational linear systems is continuing. A system of this type can be characterized in matrix form with associated input and output vectors. Through this characterization necessary and sufficient conditions on the matrices and associated vectors of two systems have been found such that these systems are equivalent.

43-1-12 FUNDAMENTAL CHARACTERISTICS OF STOCHASTIC DISCRETE-STATE SYSTEMS. Y. Sekine (Prof. Zadeh)

The objective of this research is to develop theoretical techniques for optimal operation and planning of large scale complexes, such as power systems with randomly varying elements, communication systems,

transportation systems, etc. The system is assumed to occupy one of a finite or infinite number of discrete states in its state space. The property of each state is described by two parameters, the transition probability from one state to another and the probability distribution of the duration of each state.

Based on these two quantities various probabilistic aspects of the behavior of such systems have been analyzed. Some of these are listed in the following.

1) Probability of mth entrance to jth state in the interval (t, t + dt) after the entrance to ith state at t = 0.

2) Probability of the system entering jth state in the interval (t, t + dt) for the mth time under the condition that the system was in ith state at t = 0.

3) Probability that the system is in jth state at time t, under the condition that the system entered the ith state at t = 0.

4) Probability that the system is in jth state at time t, under the condition that the system was in ith state at t = 0.

Research is now proceeding on complex systems which consist of elementary systems, each of which is defined in a similar manner to that discussed above. Particular attention is given to the treatment of transient states and recurrent states and to reduction of a complex system to an equivalent simple system (when it is possible). At present research is centered on various problems concerning the addition of a new elementary system to the existing complex system. Incidentally, the flow graph analysis, the theory of games and programming techniques are used as tools in this research.

43-1-13 A NUMBER THEORETIC APPROACH TO THE STUDY OF IRREDUCIBLE NONNEGATIVE MATRICES. B. Whalen (Prof. Zadeh)

Irreducible nonnegative matrices are a generalization of the stochastic matrices of ergodic Markov chains. Primitive matrices are the corresponding generalization of the stochastic matrices of regular Markov chains.

It is well known that the properties of these matrices can be deduced topologically. In this study it has been shown that the topological problems can be reduced to one in number theory. In this context the proofs of the results are simple and quite straightforward.

43-1-14 A NOTATION FOR n-VARIABLE DIFFERENTIAL CALCULUS. B. Whalen (Prof. Zadeh)

A notation which combines the gradient operator with the Dirac notation has been developed. This enables one to state and prove results

in differential calculus, calculus of variations and related fields in a compact but illuminating manner. This notation is expected to be helpful in the formulation and solution of multidimensional control problems.

The two additions to the standard Dirac notation are

$$\nabla_x\rangle = \begin{bmatrix} \frac{\partial}{\partial x_1} \\ \frac{\partial}{\partial x_2} \\ \vdots \\ \frac{\partial}{\partial x_n} \end{bmatrix}$$

and

$$\langle y|\nabla_x\rangle^k = \sum_{i_1,\ldots i_k} y_{i_1},\ldots y_{i_k} \frac{\partial^k}{\partial x_{i_1},\ldots \partial x_{i_k}}$$

In this notation the chain rule becomes

$$\nabla_y F(y) = \nabla_y \langle x|\nabla_x\rangle f$$

where

$$F(y) = f(x)$$

The Eulor equations become

$$\nabla_x L - \frac{d}{dt}\nabla_{\dot{x}} L = 0$$

The Hamilton equations become

$$\dot{y} = -\nabla_x H$$

$$\dot{x} = \nabla_y H$$

and the Taylor series becomes

$$f(x + h) = e^{\langle h|\nabla_x\rangle} f(x)$$

43-1-15 AN ITERATIVE SOLUTION TO TWO PERSON GAMES.
B. Whalen (Prof. Zadeh)

A very simple iteration for solving two person games has been devised. The method is not assured of convergence, but divergence can be easily detected and the iterations reset with a new initial condition. For simple games the technique is especially useful.

The procedure differs from those of Brown and Von Neumann in that it uncouples the strategy searches of the two players.

MISCELLANEOUS

02-1-17 ELECTROPHYSIOLOGICAL PROPERTIES OF THE CEREBRAL CORTEX. G.G. Furman (Prof. Susskind)

Experiments with the stimulation of the olfactory cortex of the cat were carried out with the use of chronically implanted bipolar leads for recording and stimulation. The final impulse response data was obtained by averaging hundreds of time responses so as to minimize the noise to signal ratio.

The recorded signals represent the weighted average of the hundreds of neurons in the vicinity of the electrode pair. The configuration of these neurons is known to approximate a thin layer of regularly aligned voltage dipoles. By straddling this dipole layer with differential leads double the voltage was obtained. As neuronal oscillations are predominantly of the relaxation type the appearance of exponentially damped cosine oscillations was surprising.

In order to interpret the source of the highly nonsinusoidal waveforms which were often obtained with harmonic oscillation on the single neuron level, a Fourier integral with the frequency function being a population density was used. Such a solution is, of course, possible for almost any waveform. It is only pertinent here if facts about the population distribution correlate with the resultant time functions.

It was found that a normal distribution of damped sinusoidal oscillators, with equal Q leads to a time response:

$$O(t) = K \cos\left(\omega_m t - \frac{\sigma^2 t^2}{QC_1} + \theta\right) e^{-\frac{\omega_m t}{2QC_1}} e^{-\frac{t^2\sigma^2}{2}\left(1 - \frac{1}{4Q^2C_i^2}\right)}$$

where $C_1 = \sqrt{1 + \frac{1}{4Q^2}}$, ω_m is the mean frequency, σ is the standard deviation and θ is the negative arctan of the sine and cosine frequency distributions.

Such a curve did indeed provide an excellent fit for many of the experimental curves. The changes resulting from higher stimulus intensities, for example, call for larger values of σ. Obviously, such a solution is not unique, additional experimental data on intraneuronal activity being required.

The evidence for harmonic oscillation was also developed, taking into account available single cell data and the geometry of the cortex.

This work was carried out with the cooperation of Professor of Physiology W.J. Freeman.

14-9-02 CELLULAR AND LONGEVITY EFFECTS OF MICROWAVE RADIATION. P.O. Vogelhut (Prof. Susskind)

The difficulties encountered in the instrumentation necessary for the accurate measurement of protein hydration, which were mainly due to the frequency pulling of the sample cavity, were corrected. The power incident in the analyzing cavity was split by a balanced waveguide-Tee, one branch thereof continuing towards the cavity, the other terminating in a phase-shifter and an adjustable short. In this manner a standing wave minimum can be obtained at the entrance iris of the cavity and the fact assured that the Q of the cavity will not change due to external conditions as the frequency response of the cavity is changed by introduction of the sample.

Measurements on solutions of pepsin in water were made and the data interpreted in terms of water molecules irrotationally bound to the protein surface. In the course of these experiments it was found that simultaneous exposure of the sample contained in the microwave cavity to electromagnetic waves in the frequency range of 100 to 500 megacycles produced changes in the sample that could be detected in changes of the cavity response curve at microwave frequencies.

To investigate the significance of these findings the theory of the experiment, which could not account for the observed phenomenon, had to be reexamined.

To arrive at a better theory of action of electromagnetic radiation on the constituents of living matter a different view was taken of the protein solution. The protein was assumed to constitute an impurity of various amounts in the water system, which was described in a semistatistical manner as a crystalline system with various compartments of four to zero bonded water molecules. The lattice defects that were taken into account were of two kinds: ionic defects and orientational defects. Due to the existence of these defects water in its pure state exhibits two distinct dispersion regions, one at about 1 megacycle and another at about 10 kilomegacycles. As impurities are introduced into such a system the dispersion frequencies shift. Since the hydration experiments are conducted at the orientational defect dispersion region and this response was influenced by irradiation at the ionic defect dispersion region, a theoretical connection can be seen between these two processes, namely, the fact that currents can flow on the surface of the protein molecules due to ionic defects and orientational defects and their mutual cooperation.

The above investigation is being continued with pepsin while it is operating as a catalyst.

01-1-62 DEVELOPMENT OF VARIABLE SPEED INDUCTION MOTORS. S.A. Nasar (Prof. Saunders)

The experimental work on the assembly and testing of the LOGMOTOR (Variable Pole Pitch Induction Motors) has been about completed. Final

tests are being conducted to obtain a speed-torque characteristic without cusps. As a consequence of the work to date the development of a motor whose coil connections are externally and continuously controlled to produce a specified speed-torque characteristic is under way. This new aspect of the project is in the initial stages.

01-1-41 HOMOPOLAR TORQUE CONVERTER. (Profs. Colgate, Koch and Saunders)

The copper-steel rotors of the device have been replaced by solid steel rotors. With mercury as collecting material, running tests of the motor section of the machine have been made. In these the expected operating speed was not reached. The distribution of the collecting material in the well (mercury) at various speeds has to be improved. Appropriate studies will be made.

01-1-63 MACHINE END-CONNECTION LEAKAGE. S.A. Stone (Prof. Saunders)

This is a new project looking toward a reevaluation of the equations used for the end-connection leakage in electrical machinery. Effort thus far has been directed at a literature search. The works of P. Hammond and C.J. Carpenter in England on the method of images have been surveyed. This approach holds some promise. Other attacks obtaining a direct solution to Laplace's equation in the end region are also under study. Some work has been done by R.T. Smith and U.B. Honsinger in this country using this approach. Upon completion of the analysis of past work attempts will be made to formulate, solve and check experimentally, if possible, the end-connection leakage in some machine configurations.

01-0-30 TRANSISTORIZED DIFFERENTIAL CIRCUIT BREAKERS FOR UTILIZATIONS CIRCUITS. (Prof. Dalziel)

Experimental work during this period has been devoted to investigating transistor amplifying and switching circuits suitable for application to differential circuit breakers. Circuit breakers have been designed and perfected. Lectures and demonstrations of the circuit breaker have been held in Newark and Berkeley, California, Geneva, Switzerland and Tokyo, Japan.